UN RS
OF ICHIGAN
SEP 24 1951
ENGINEERING
LIBRARY

APPLIED MECHANICS Reviews

A CRITICAL REVIEW OF THE WORLD LITERATURE IN APPLIED MECHANICS AND RELATED ENGINEERING SCIENCE

REVS. 3442-3745 VOL. 4, NO. 9 SEPTEMBER 1951

GENERAL

Theoretical and Experimental Methods.. 491
Mechanics (Dynamics, Statics, Kinematics) 494

MECHANICS OF SOLIDS

Gyroscopics, Governors, Servos..... 498
Vibrations, Balancing..... 499
Wave Motion, Impact..... 499
Elasticity Theory..... 500
Experimental Stress Analysis..... 501
Rods, Beams, Shafts, Springs, Cables, etc. 502
Plates, Disks, Shells, Membranes..... 503
Buckling Problems..... 504
Joints and Joining Methods..... 504
Structures..... 505
Rheology (Plastic, Viscoplastic Flow).... 508
Failure, Mechanics of Solid State..... 509
Material Test Techniques..... 509
Mechanical Properties of Specific Materials..... 510
Mechanics of Forming and Cutting..... 513

MECHANICS OF FLUIDS

Hydraulics; Cavitation; Transport.... 514
Incompressible Flow: Laminar; Viscous.. 514
Compressible Flow, Gas Dynamics..... 517
Turbulence, Boundary Layer, etc..... 522
Aerodynamics of Flight; Wind Forces... 523
Aeroelasticity (Flutter, Divergence, etc.). 526
Propellers, Fans, Turbines, Pumps, etc.... 526
Flow and Flight Test Techniques..... 527

HEAT

Thermodynamics..... 528
Heat and Mass Transfer..... 530

MISCELLANEOUS

Acoustics..... 532
Ballistics, Detonics (Explosions)..... 533
Soil Mechanics, Seepage..... 533
Micromeritics..... 534
Geophysics, Meteorology, Oceanography..... 535
Lubrication; Bearings; Wear..... 536
Marine Engineering Problems..... 536

Communications, 491

Recent Trends in Theories of Plates and Shells, by W. Flügge, 489

Published Monthly by
THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS
at Easton, Pa., and edited by
Midwest Research Institute with the co-operation of Linda Hall Library

APPLIED MECHANICS Reviews

Under the Sponsorship of

THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS • SOCIETY FOR EXPERIMENTAL STRESS ANALYSIS • INSTITUTE OF THE AERONAUTICAL SCIENCES • AMERICAN INSTITUTE OF PHYSICS • AMERICAN MATHEMATICAL SOCIETY • AMERICAN SOCIETY OF CIVIL ENGINEERS • AIR RESEARCH AND DEVELOPMENT COMMAND • THE ENGINEERING FOUNDATION • THE ENGINEERING INSTITUTE OF CANADA • THE INSTITUTION OF MECHANICAL ENGINEERS • MIDWEST RESEARCH INSTITUTE • OFFICE OF NAVAL RESEARCH

EDITOR	Martin Goland
EDITORIAL ADVISERS	T. von Kármán, S. Timoshenko
EXECUTIVE EDITOR	D. R. Mazkevich
ASSOCIATE EDITORS	J. C. Shipman, J. J. Jaklitsch, K. Zarankiewicz (Poland), Isao Imai
ASSISTANT EDITORS	M. Garrett, H. K. Wilgus, S. Lees
EDITORIAL ASSISTANTS	M. Holloway, I. Gr
PUBLICATIONS MANAGER	S. A. Tucker
OFFICERS OF ASME	J. Calvin Brown, *President* J. L. Kopf, *Treasurer* C. E. Davies, *Secretary*
ASME MANAGING COMMITTEE	G. B. Pegram, *Chairman* H. L. Dryden, R. E. Peterson, LeVan Griffis, J. M. Lessells, L. H. Donnell, *ASME Applied Mechanics Division*
ADVISORY BOARD	R. E. Peterson (ASME), *Chairman* R. D. Mindlin (SESA), *Secretary* K. O. Friedrichs (AMS), G. R. Rich (ASCE), K. S. M. Davidson (IAS), F. V. Hunt, J. J. Green (EIC), H. W. Swift, E. Bromberg (ONR), F. W. Bubb (ARDC)

Editorial Office: **APPLIED MECHANICS REVIEWS, Midwest Research Institute, 4049 Pennsylvania, Kansas City 2, Mo., U. S. A.**
Subscription and Production Office: **The American Society of Mechanical Engineers, 29 West 39th St., New York 18, N. Y., U. S. A.**

HOW TO OBTAIN COPIES OF ARTICLES INDEXED: Photocopy or microfilm copies of all articles reviewed in this issue will be provided on request. Orders should specify the number and issue of APPLIED MECHANICS REVIEWS; should be addressed to APPLIED MECHANICS REVIEWS, Midwest Research Institute, 4049 Pennsylvania, Kansas City 2, Mo., and be accompanied by a remittance to cover costs. Photocopy costs are 25¢ for each page of the article photocopied; minimum charge $1; Microfilm costs include service charge of 50¢ per article, plus 3¢ per double page; minimum order, 75¢. *(Applicant assumes responsibility for questions of copyright arising from this copying and the use made of copies. Copyright material will not be reproduced beyond recognized "fair use" without consent of copyright owner.)*

APPLIED MECHANICS REVIEWS, September 1951, Vol. 4, No. 9. Published monthly by The American Society of Mechanical Engineers at 20th and Northampton Sts., Easton, Pa., U. S. A. The editorial office is located at the Midwest Research Institute, Kansas City 2, Mo., U. S. A. Headquarters of ASME, 29 West 39th St., New York 18, N. Y., U. S. A. Cable address: "Dynamic," New York. Price $1.50 per copy, $12.50 a year; to members of ASME and co-operating societies $0.75 per copy, $9 a year. Extra postage to countries outside Pan American Union, $1.50, to Canada, $0.75. Changes of address must be received at Society headquarters four weeks before they are to be effective on the mailing list. Please send old as well as new address . . . . By-Laws: The Society shall not be responsible for statements or opinions advanced in papers or printed in its publication (B13, Par. 4). Entered as second-class matter, January 11, 1948, at the Post Office at Easton, Pa., under the Act of March 3, 1897. Copyrighted, 1951, by The American Society of Mechanical Engineers

APPLIED MECHANICS REVIEWS

VOL. 4, NO. 9 — MARTIN GOLAND *Editor* — SEPTEMBER 1951

RECENT TRENDS IN THE THEORIES OF PLATES AND SHELLS

W. FLÜGGE

STANFORD UNIVERSITY, STANFORD, CALIFORNIA

THE THEORY of plates and the theory of shells have both had their high tide in the 1920's, when civil engineers became aware of what could be done with them. Since that time development has slowed down, but there is still much activity in both fields, as one may easily see by a glance at the monthly issues of these Reviews.

In the present article an attempt will be made to trace the lines which have been followed by the research of the last decade, and at the end a few papers will be listed which may be considered typical illustrations for the topics treated.

PLATES

Engineers became plate-minded when they recognized the advantages which the theory of plates offers in the design of floor slabs. Many papers appeared, most of them concerned with rectangular plates. On the crest of the wave stands Nadai's book (1925), and some time after its ebbing the results were consolidated in Timoshenko's book (1940). Since that time activity along the old line has still continued and papers are appearing currently which either try to find simpler solutions for the most frequently used cases, or to add new details to the existing stock of knowledge. Topics of special interest are plates on point supports, loads distributed over a small area (1),* mixed boundary conditions, and plates of odd shape, e.g., parallelograms (2). Among the methods used, the application of Fourier series and Fourier integrals prevails. There is a definite trend away from the energy method which yields a good general idea of the stress distribution but is unreliable in the details and may fail to show points of stress concentration. The same holds for the once favored method of finite difference equations which replaces the differential equation by, say, a dozen linear equations and draws from them a somewhat blurred picture of the stress pattern. Its place has been taken by the relaxation method which permits a much larger number of check points to be handled easily. The method of splitting off the singularities seems to be not yet as popular as it should be. Skillfully combined with relaxation, it may become the everyday tool for the numerical solution of most plate problems.

INFLUENCE FUNCTIONS

When bridge engineers began to use concrete floor slabs on steel highway bridges, a vivid interest in influence diagrams arose. Just as the common influence lines, they are based on the reciprocity law of Maxwell and Betti. Each influence function is given by the deflections of a suitably loaded plate. When the deflection at a certain point is sought, the plate must be loaded by a concentrated force at this point; for the influence function of a slope the fictitious load must be a couple; and for the more interesting influence diagrams of bending and twisting moments the load is a higher singularity. In this field the method of singularities has been widely used (3), which starts the solution with a simple expression having the correct singularity, and then satisfies the boundary conditions by adding a continuous solution which may easily be computed from a very few terms of a series or by relaxation.

NONLINEAR PROBLEMS

The general interest in nonlinear problems, which characterizes present activities in some fields, has also reached plate theory. Here the large-deflection theory bridges the gap between the common plate theory and the theory of thin prestressed membranes, and it is closely connected with the problem of postbuckling behavior of plates, which represents merely the particular case of zero lateral load.

The circular plate (4) has been treated frequently and has been used as a kind of testing ground for the development of mathematical methods; more practically minded research workers have tried to tackle the problem of square and rectangular plates (5).

SANDWICH PLATES

When speaking of sandwich plates, one usually thinks of buckling and wrinkling. But the bending problem of sandwich plates also has been given some consideration (6). It raised the question of what influence the shear deformation has on the deflection and on the distribution of the stresses in the plate (7); and it also revived interest in a critical discussion of Kirchhoff's boundary condition (8).

MEMBRANE THEORY OF SHELLS

The great days of shell theory occurred in the early 1930's when Dischinger developed revolutionary ideas and built large shell

* Numbers in parentheses refer to the Bibliography at the end of the paper.

EDITOR'S NOTE: The ninth in a series of articles, contributed to APPLIED MECHANICS REVIEWS by international authorities, surveying important topics in applied mechanics.

roofs of various shapes. Close to the end of this period the writer's book appeared (1934), consolidating the knowledge of that time. In later years, the development has slowed down but steady progress has been made in many fields.

STRESS FUNCTION

When Pucher published (1934) the paper on the stress function which now bears his name, it appeared to be just a very general approach to the differential equation of the membrane theory of shells. But since Southwell developed his relaxation method, it seems that the combination of the two will be a powerful tool for the numerical solution of many shell problems. So far, only a few attempts in this direction have been published (9) and much remains to be done to make the method ready and safe for routine application.

SHELLS OF NEGATIVE GAUSSIAN CURVATURE

The membrane forces in thin shells may be derived from a second-order partial differential equation. When the shell has positive Gaussian curvature, this equation is of the elliptic type; but when the curvature changes sign, the equation becomes hyperbolic and its solution may show the well-known discontinuities. In the mid-1930's French engineers met with such discontinuities in hyperbolic paraboloids and they used them for a simple geometric approach (10) which opened the field to those engineers who are not familiar with partial differential equations. Later the writer investigated the one-sheet hyperboloid (11) and pointed out that the discontinuities in the membrane stresses are such that the physical reality of the solution is more than doubtful, but that the geometric approach may be very useful in such cases where the result is free from discontinuities.

SINGULARITIES

When at some point the solution of a shell problem yields infinite membrane forces, this indicates that the underlying assumptions are not adequate; but it does not necessarily mean that the solution could not be trusted outside the immediate vicinity of the singular point. A singularity may be caused by the shape of the shell or by the application of a concentrated load.

The first kind can best be studied at the apex of a shell of revolution, and there Truesdell (12) found the surprising result that in pointed shells all higher harmonics of the homogeneous solution are singular. A check on an example, however, has shown that the two independent solutions which exist for every harmonic may be chosen in such a way that their singularities are very different in strength, and it may be assumed that the weaker one is such that the bending rigidity will quell it entirely.

A point load is always a simplified representation of a load which is really distributed over a small area, and outside this area the stress pattern derived from a point load may be expected to come close to reality. Until recently, point loads have only been considered at the apex and at the edge of shells of revolution. A new idea was introduced by Martin (13) when he used Mercator's projection to transform the membrane equations of a spherical shell into relations in a complex plane. He found rather simple expressions for the internal forces in a sphere which carries a radial or tangential load at an arbitrary point and is supported at the poles by the necessary forces and couples. With the help of an affine transformation these solutions may be adapted easily to an ellipsoidal shell, but it is still an open question whether similar solutions can be found for shells of other shapes.

NONLINEAR THEORY

It is a well-known fact that the differential equations of the membrane theory are of a very low order and that, therefore, their solutions do not have enough free constants to be adapted to all reasonable boundary conditions. As far as solutions of the bending theory are known, they confirm the membrane solutions as good approximations and provide, additionally, a number of localized stress systems which may be used to satisfy those boundary conditions which the membrane theory cannot take care of.

This line of thought, now commonly used in shell theory, was supplemented by an essential new element when Bromberg and Stoker (14) investigated the large-deflection membrane theory of spherical shells. They found that the nonlinear equations are of higher order than are the linear equations so that their solutions may satisfy more boundary conditions. In the major part of the shell the internal forces may be the same as in the linear theory, and only in a boundary zone near the edge are they modified in such a way as to meet all conditions.

BENDING THEORY OF SHELLS

The stresses found from the bending theories of bars, plates, and shells are not exact solutions of the three-dimensional theory of elasticity. They approximate these (mostly unknown) solutions better the thinner the bar, plate, or shell is; but somewhere there is always a contradiction in an unimportant condition of compatibility.

In the case of a bar, the assumption that plane cross sections remain plane may easily be motivated, and from there the corresponding assumption in the theory of plates may be derived. In both cases the kinetic assumption is identical with the assumption that stresses are distributed linearly across the thickness. In shells the two assumptions are not identical, and since both lead to rather involved relations, a critical analysis is desirable. This problem has recently met with much interest. The paper mentioned in the bibliography (15) may serve as an example. Instead of hiding the discrepancy at an invisible place, the authors judge the different theories according to the power of the wall thickness t, up to which they are correct. The membrane theory, of course, ranks lowest, and the equations of the bending theory are put in such a form that they are good up to t^3, but not in excess of this.

BIBLIOGRAPHY

In the following list we give only a small selection of papers which may be considered typical for the trends mentioned in this article. No attempt toward completeness, even in the high lights, has been made. For more information, the *Zentralblatt für Mechanik*, the APPLIED MECHANICS REVIEWS, and the reference lists of the following papers should be consulted.

1 Westergaard, H. M., New formulas for stresses in concrete pavements of airfields, AMR **1**, Rev. 61.

2 Lardy, P., Die strenge Lösung des Problems der schiefen Platte, AMR **3**, Rev. 436.

3 Pucher, A., Die Einflussfelder des Plattenstreifens mit zwei eingespannten Rändern, AMR **4**, Rev. 1979.

4 Reissner, E., On finite deflections of circular plates, AMR **3**, Rev. 1244.

Federhofer, K., Die dünne Kreisringplatte mit grosser Ausbiegung, *Z. angew. Math. Mech.* **24**, 189–194, 1944.

5 Wang, C.-T., Nonlinear large-deflection boundary-value problems of rectangular plates, AMR **1**, Rev. 608.

6 Libove, C., and Batdorf, S. B., A general small-deflection theory for flat sandwich plates, AMR **3**, Rev. 1245.

7 Reissner, E., On bending of elastic plates, AMR **1**, Rev. 59.

8 Friedrichs, K. O., The edge effect in the bending of plates, AMR **2**, Rev. 990.

9 Flügge, W., Das Relaxationsverfahren in der Schalenstatik, AMR **4**, Rev. 3192.

10 Aimond, F., Étude statique des voiles minces en paraboloïde hyperbolique travaillant sans flexion, *Publ. int. Assn. Bridge Struct. Engng.* **4**, 1–111, 1936.

11 Flügge, W., Zur Membrantheorie der Drehschalen negativer Krümmung, AMR **2**, Rev. 848.

12 Truesdell, C., The membrane theory of shells of revolution, *Trans. Amer. math. Soc.* **58**, 96–166, 1945.

13 Martin, F., Die Membrankugelschale unter Einzellasten, AMR **3**, Rev. 2251.

14 Bromberg, E., and Stoker, J. J., Non-linear theory of curved elastic sheets, *Quart. appl. Math.* **3**, 246–265, 1945.

15 Hildebrand, F. B., Reissner, E., and Thomas, G. B., Notes on the foundations of the theory of small displacements of orthotropic shells, AMR **3**, Rev. 453.

Communications

Concerning AMR 4, Rev. 1953: K. H. Swainger, Finite elastic straining.

Reviewer is incorrect that tangential contraction occurs at $\theta = 30°$ on the unstressed hole and that strains occur at unstressed corners of cantilever. Theory gives zero strain at these *points* while figures are too small to allow visual estimate of strain at these points. Author thinks reviewer has overlooked that strain equations apply to points in the orthogonal transformation of the deformation problem. K. H. Swainger, England

Concerning Rev. 3458 in this issue.

It should be pointed out that the book in question is intended to be an introduction to the subject. To cover the material in the manner the reviewer wishes would increase its size to a point where it would not be suitable for the usual undergraduate course. The reviewer's criticisms, many of them controversial, would appear to apply to all current American texts. The wide acceptance of this text in its past four editions shows the merit of its organization which has not been changed in the present revision. It is perhaps unfortunate that a teacher was not chosen to review this book since it is primarily a text. A. H. Church, USA

Concerning Rev. 3618 in this issue.

The author does not fully agree with the opinions expressed in the last three paragraphs of the review.

1 The perturbation considered does not involve any contraction of the stream tube area. Therefore, it is not the most general perturbation. As, however, a system with many degrees of freedom is unstable as soon as there exists one steadily increasing perturbation, consideration of these more general perturbations cannot render the argument inconclusive.

2 The initial state of the flow in this analysis is a transonic flow in which the internal and translational temperatures are equal. In the transition range of a thickened shock wave the flow is subsonic and there is no thermal equilibrium. Therefore, these cases are certainly not identical.

3 It is quite true that, strictly speaking, the statement quoted has no meaning because any reference to a coordinate system is lacking. The same remark can be made with respect to the usual statement that in a steady flow shocks can occur only where the flow is supersonic. A less inexact way to formulate these questions is: Do there exist boundary conditions which make the considered type of flow a part of a steady, unique, and stable solution of the equations of motion? Or, in more physical terms: Is it possible to obtain and keep the considered flow pattern at a given phase by suitable modeling of the duct and attaining the steady state in a suitable way, without material interference with the character of the flow as calculated by the incomplete theory? In the opinion of the author, the results obtained make it clear that, in the case of transonic steady flow, this stabilization without perturbance is not possible. It is not claimed that a rigorous mathematical proof is given.

L. J. F. Broer, Holland

Theoretical and Experimental Methods

(See also Revs. 3477, 3478, 3491, 3594, 3670)

3442. Bernard J. J., and Siestrunck, R., On the utilization of successive approximations in the determination of certain aerodynamic potentials (in French), *Rech. aéro.* no. 17, 3–8, Sept.-Oct. 1950.

Author discusses the partial differential equation div (h grad φ) $= 0$, in which $\varphi = \varphi(x, y)$ is regular at infinity and on an interior boundary (C) satisfies the boundary conditions of the Fourier type, $(d\varphi/dn) = \mu(P) + B(P)$. Here h is a positive, slowly varying function $h(x, y)$, and μ and B are given functions on (C). Author considers only the case where, on the boundary, h satisfies the additional condition $dh/dn = 0$.

When μ is positive, a practical solution can readily be obtained by the methods of electrical analogy. When μ is negative, however, the electrical arrangement becomes much more complicated, and, consequently, a method of successive approximations has been derived by which the problem can be treated with simpler electrical circuits.

The convergence of the method of successive approximations is discussed first for the case where μ is a constant and then for the case where μ is a variable. The proof of convergence in the former case is based on general properties of Fredholm integral equations, and a simple criterion for convergence is derived. This case is illustrated by application to the potential of wings of finite span. In the more general case, a considerably more complex criterion for convergence is derived and applied to discuss the potential of contrarotating propellers.

Louis Landweber, USA

3443. Watanabe, M., and Miita, J., Application of the differential analyzer and its performance characteristics (in Japanese), *Rep. Inst. Sci. Technol. Tokyo* **3**, 3–4, 65–69, Mar.-Apr. 1949.

Characteristics of the differential analyzer of Bush type, which was constructed in 1945 by Dr. T. Sasaki and improved by the present authors, are described. Authors present a practical method for determining the optimum values of scale factors and gear ratios to obtain solutions with high accuracy, giving as an example the solution of equation of a certain pendulum motion. Accuracy of the integrators and the propagation of errors caused by some other factors, such as inaccuracies in curve tracing and backlashes of connecting gears, are analyzed both theoretically and experimentally. Accuracy of the machine seems to be sufficient for most practical purposes.

Humio Tamaki, Japan

3444. Gerhardt, J. R., A simple integrating device for the routine statistical analysis of recorded data, *Trans. Amer. geophys. Un.* **31**, 2, 183–186, Apr. 1950.

Paper describes a mechanical-electrical integrating device used to speed up the calculations of the mean and standard deviation of a time-varying quantity, e.g., temperature. Ten clocks record the time intervals during which the temperature has been greater than given preset temperatures. Response lag is about 4 sec, approximately that of the sensing element.

F. J. Maginniss, USA

3445. Morse, P. M., and Kimball, G. E., Methods of operations research, New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd.; 1951, 158 pp. $4.

"Operations Research is a scientific method of providing executive departments with a quantitative basis for decisions regarding the operations under their control." Thus do the authors define the new applied science for which this is the first introductory book. The field is largely quantitative analysis of operational problems with a view toward optimization, leaning heavily upon mathematical statistics, variational calculus, and the Theory of Games. The authors stress the practical nature of Operations Research, i.e., the intimate liaison with the executive function, as well as the urgency of useful results. It is also pointed out that the operations researcher makes no executive decisions but provides such pertinent information as can be expressed quantitatively. Although similar techniques had been used in the past, organization into a broad discipline was started, apparently, by British radar workers during World War II. Used largely for military problems up to the present time, Operations Research is obviously applicable to a broad range of problems in industry, government, etc. The military birth shows in the chapter headings: 1. Introduction; 2. Probability; 3. The use of measures of effectiveness; 4. Strategical kinematics; 5. Tactical analysis; 6. Gunnery and bombardment problems; 7. Operational experiments with equipment and tactics; 8. Organizational and procedural problems.

The first two headings are self-explanatory. The third chapter is concerned with efficiency criteria for search operations and for miscellaneous equipment performance, as well as the notion of "exchange rates" between two warring parties. The fourth is largely devoted to formulation of differential equations ("Lanchester's equations") giving simple hypothetical descriptions of the exchange rate problems in warfare, including mention of the "minimax principle" of the Theory of Games. This is a two-parameter variational problem in which one is adjusted to maximize a function while the other is simultaneously adjusted to minimize the same function, including the interaction.

The fifth chapter deals with optimization of tactics, countermeasures and counter-countermeasures, including search and bombardment problems, while the sixth covers ballistic statistics including the effects of both aiming and ballistic dispersions on procedures for maximization of destructive efficiency. The seventh and eighth chapters are descriptive except for the statistics of gunnery accuracy.

Reviewer's remarks: In spite of a certain lack of integration (attributable to both multiple authorship and to the newness of the field), this book is extremely stimulating. The text is well written and has a profusion of detailed examples, mostly military. Pedagogically, some specific quantitative sections could probably be improved. (Examples: The unorthodox method for introducing the concept of probability distribution function; the lack of systematic explanation of the interesting Poissonian similarity between systematic search and random search, e.g., bombardment). But on the whole, the excellence of the discussion more than overbalances these minor shortcomings.

No mention is made of the applicability of Information Theory to operations work, although one remarkable example came up during World War II [Shannon, "Communication theory of secrecy systems," *Bell System tech. J.* 28, Oct. 1949].

In placing overwhelming textual emphasis upon the practicality and urgency of work in Operations Research, the authors slight the importance of basic studies which could be carried out under drastically idealized conditions. Yet much of chapter three is just such an idealized study—for which they apologize! When Operations Research enters the universities, it should probably concern itself with basic matters, leaving the practical and urgent questions to the military and industrial-operations workers.

The reviewer recommends this book highly to anyone with more than a passing interest in the effectiveness of organizations or of any operations under human control.

Stanley Corrsin, USA

3446. Baticle, E., Interpretation of results of tests of a sample (in French), *Génie civ.* **77,** 13, 246–248, July 1950.

Article relates to a statistical method of sampling industrial supplies. Author derives simple formulas for a posteriori probability of quality parameter. It is claimed that these give, from minimum number of trials, the law of probability of x successes in n trials when one makes n or more trials of which K are unfavorable.

Deduction that $F(x, p) = Pr\{X < x\}$ is also a distribution function in variable P may be strongly questioned. Modern developments in testing of statistical hypotheses, e.g., fiducial limits, are ignored.

Four typographical errors in fourth section do not simplify the exposition.

S. Kirkby, England

3447. Wald, Abraham, Statistical decision functions, New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd.; 1950, ix + 179 pp. $5.

In this book Professor Wald has brought together virtually all that is known concerning the theory of statistical decision. Results which up to now were available only in journal articles are related one to the other and made part of a coherent theory. In addition, Professor Wald presents some new material which had not been published previously. Wald has laid down the general framework of a decision theory, has proved a number of key theorems, and has indicated that a vast unexplored territory lies before us. With his untimely passing (Professor Wald and his wife were killed in an airplane accident in India on December 13, 1950) those workers who remain will return time and again to this book for inspiration and guidance.

Perhaps Wald's greatest contribution is that he focused attention on the really central aim of the statistician and statistical theory, namely, to make the best possible decisions on the basis of information furnished by observations made on a random process. This statement is meant to encompass the efficient design of experiments where the object is to run experiments in such a way as to get the maximum amount of information per observation. It includes the problem of making the "best" estimate of some unknown parameter(s) from a set of observations. It includes the problem of minimizing in some sense the chances of making a wrong choice between two or more hypotheses when all there is to go on are a number of observations following some probability law. In its all-inclusiveness, this theory becomes of central importance to physical, biological, and social scientists, to engineers, and to all those who are continually grappling with the problem of how to design experiments better, of how to get the most out of their data, and of how to take an action on the basis of the observations which is best in some reasonable sense.

To R. A. Fisher, the eminent British statistician, and his school do we owe the great strides which have been made in the scientific design of experiments. His classic work, "Design of Experiments," should be required reading for all who are experimenters, whether their purpose be to wrest another secret from Nature, to find a cure for any one of the terrible diseases which affect man and other living things, or to find a better way of making a screw. To Jerzy Neyman and Egon Pearson and indeed to many statisticians both here and abroad do we owe the beautiful theory of testing hypotheses and estimation. In this theory Neyman and Pearson made explicit the ideas of "best" estimates and "best"

tests, introduced various criteria for "bestness," and developed methods for finding best tests when they existed. They introduced the ideas of errors of first and second kind, acceptance (or rejection) regions, power curves, and made people aware that they should choose tests which, as frequently as possible, lead to making a right decision. If, for instance, we wish, on the basis of a number of tests, to decide (*a*) whether to say that a patient has TB, or (*b*) to say that he does not have TB, it is clearly desirable to use the data in such a way that, on the one hand, we do not send a sick person out into the world without treatment or, on the other, commit well persons to a TB sanitarium. Unfortunately, it is not possible on the basis of limited observations to avoid these kinds of errors. In fact, one cannot simultaneously make both of these errors small without paying for this assurance by increasing the number of observations. In the Neyman-Pearson theory a method is developed for handling these kinds of problems. These ideas are perhaps best known to engineers by their application (an independent and to some extent prior development) by Harold Dodge and H. G. Romig of the Bell Telephone Laboratories. In this context errors of the first and second kind become, on the one hand, rejection of good lots (producers' risk) and on the other, acceptance of bad lots (consumers' risk).

In the light of earlier work by Fisher, Neyman, Pearson, and many others, Wald's great contribution is that he introduced a new element into the design of experiments and testing of hypotheses. He recognized that while it is important to make the probability of a wrong decision and hence wrong action as small as possible, it is essential to consider the possible loss due to taking the wrong action on the basis of the evidence. If one defines with Wald the idea that the risk of a procedure is the expected cost of taking observations plus the expected loss, it is clear that it is desirable in some sense to choose that decision procedure which minimizes the risk. Wald proposes a procedure known as minimax, which is to choose that decision rule which has the smallest maximum risk; others (including Wald) have suggested other possible methods for choosing a "best" decision procedure, e.g., to choose that procedure for which the average risk is as small as possible. While arguments may be presented pro and con for various definitions of best decision procedures, it is clear that such problems should be of basic concern to experimenters and to those charged with making possibly high policy decisions on the basis of collected information.

Perhaps the greatest difficulty in applying this theory (and hence an area in which much work should be done) is that Wald assumes that cost of experimentation, losses, and hence risks can all be measured in terms of money. While this is a useful assumption to make in building up a purely mathematical theory, it is apparent that there are many practical problems where the cost of a wrong decision could not possibly be assessed in purely monetary terms. It may be a cliché, but it is still to the point to ask how much is a human life worth, if our decision happens to be the erroneous one of letting a dangerously contaminated consignment of drugs go out on the market? Similarly, how does one measure in monetary terms the wrong decision not to manufacture a drug which turns out some years later to be really an effective cure for some disease? Clearly these problems become most critical in the area of the social and biological sciences, although they are surely present in all problems, and consequently some general theory of value will have to be brought in. Value judgments involving at least a ranking of the objectives of an experiment will have to be made if one wishes to apply Wald's ideas in the real world.

It is always unpleasant for a reviewer to have to point to a blemish in a book. It is particularly so in this case, since this work will be a classic no matter what its minor faults. The book is difficult to read. This is true for the professional statistician and is a hundredfold true for the natural scientist, the social scientist, the medical man, the engineer. Results are given so abstractly and the treatment is so complicated that I think many readers will simply give up in despair. There should have been more motivation and more could have been accomplished by not trying to be so general as to obscure essential ideas. This criticism is directed not only at this book, but also at all those who feel that it is not necessary to motivate their results and to take the reader into their confidence.

In closing, the reader's attention is directed to a number of articles which explain in a simple way some of the basic notions in Wald's theory of decision functions. These are: Friedman, M., and Savage, L. J., "The utility analysis of choices involving risk," *J. Political Economy* **56**, 279–304, 1948; Sprowls, R. Clay, "Statistical decision by the method of minimum risk: An application," *J. Amer. statist. Assn.* **45**, 238–248, 1950; Savage, L. J., "The theory of statistical decision," *J. Amer. statist. Assn.* **46**, 55–67, 1951. It is also recommended that the interested reader study carefully chapters 1 and 5 of Professor J. Neyman's recent book "First course in probability and statistics" [New York, Henry Holt and Co., 1950]. Benjamin Epstein, USA

3448. Guest, P. G., Estimation of the error at a point on a least-squares curve, *Austral. J. sci. Res. Ser. A*, **3**, 173–182, 1950.

Author treats problem of estimating the errors in the predicted values obtained by fitting a least-squares polynomial to a number of equally spaced observations on a random variable. A number of curves and tables summarizing the results are given. Calculations are facilitated by use of orthogonal polynomials.

Benjamin Epstein, USA

3449. Finch, J. K., Engineering and western civilization, New York, McGraw-Hill Book Co., Inc., 1951, x + 397 pp. $5.

This book is both a history of engineering and an analysis of its effect on civilization as we know it. The introduction and first six chapters describe briefly but interestingly the beginnings, development, and increase in stature of engineering and engineering knowledge. These chapters point up the important part that engineers and engineering played in the rise of Western civilization.

From chapter seven through chapter thirteen, the author outlines the rise in engineering science and goes on to describe the scope and activities of engineering as it is today in the fields of transportation, material, power, industry and manufactures, communications and electronics, and construction. This review of modern engineering and its works is very comprehensive, but is too much a repetition of well-known facts. Here, too, the author's own words as given in the foreword to the bibliography might be kept in mind. "A word of caution seems desirable. As a result, the brief historical statements given in many engineering papers and texts are too often incorrect or misleading."

In chapter 14, entitled "Engineering and Western civilization," the author gets to the important and most interesting part of the book. As he puts it, "The future of engineering is thus tied up with the future evolution of our Western way of life." With statistics and examples, he points out how engineers and engineering have improved the lot of mankind and how, at the same time, our civilization has come to depend upon the accomplishments of the engineer and to need the engineering method for its future progress.

To the reviewer, the most important part of the book is the clear-cut way in which the author presents the economics of our modern industrial civilization. Every engineer, particularly those in managerial positions, should read this portion of the book. More important, if our labor leaders and businessmen also

read this portion, it would undoubtedly help toward better relations and industrial harmony.

In a book of only 300-odd pages, the author can touch only briefly on the many interesting achievements of mankind. In reading the book, one's curiosity is constantly aroused and the net result is a whetting of one's appetite for more information on this important phase of human history. To help the reader in this, the author has prepared a most extensive bibliography, broken down by chapters and subjects. Altogether, this is a very worth-while book and should prove a valuable addition to any engineer's library. A. O. White, USA

3450. Heavens, O. S., Measurement of the thickness of thin films by multiple-beam interferometry, *Proc. phys. Soc. Lond. Sec. B*, **64,** part 5, 377B, 419–425, May 1951.

By multiple-beam interferometry according to Tolansky (1948), thickness measurements on films of silver and lithium-fluoride between 120 A and 1500 A have been made with five different metals as reflecting layers. The accuracy is 15 A because of small thickness variations of optically polished glass. When fresh, all metals give the same results; Ag and Al yield sharpest fringes due to high reflectivity; aging effects are observed only in the case of Cr and amount to 15–20% within a few days.

Margot Herbeck, Germany

3451. Withers, R. M. J., Wire-drawing die bores. Survey of methods for examining their profiles, *Metal Treatm.* **18,** 68, 191–194, May 1951.

Author describes some of the optical and mechanical methods and instruments developed specifically for measurement or gaging of wire-drawing die bores of small diameter.

From author's summary

3452. Burge, E. J., and Jackson, L. C., The thickness of the helium film, *Proc. roy. Soc. Lond. Ser. A*, **205,** 1081, 270–285, Feb. 1951.

A description is given of an optical method for determination of the thickness of the helium II film. Linearly polarized light is reflected from a stainless-steel mirror, upper part of which is coated with a layer of barium stearate one molecule thick and lower part with a similar layer three molecules thick. The reflected light passes through a mica compensating plate and a nicol prism. Adjustment of the mica plate and nicol gives equality of illumination on the "1" and "3" areas. If, now, a film of helium II covers the mirror, the nicol must be rotated to restore equality of illumination. The rotation is a measure of the helium film thickness, the relation between the two quantities being calculated in terms of the angle of incidence and the optical constant of liquid helium, barium stearate, and stainless steel. Observed thickness at any given height above the liquid helium was found to be nearly independent of the temperature between 1.1 and 2.18 K, but then decreased rapidly to zero at the λ-point. In the formula $d = k/h^{1/z}$ for the thickness d at height h cm for any given temperature, the value of z varies from 3.5 at 1.1 K to 2.5 at 2.1 K. The thickness at 1 cm and 1.5 K is provisionally given as 1.9×10^{-6} cm. From authors' summary

3453. Ishiguro, K., Numerical tables for the phase and intensity measurements of metallic film I, II, *J. phys. Soc. Japan* **6,** 1, 1–19, Jan.-Feb. 1951.

In optical investigations of metallic films, it is aimed to determine the refractive index and film thickness from measurements of transmission, reflection, and rate of phase shift; this requires very troublesome calculations. Calculation in the reverse direction, namely, the determination of the transmission, etc., when the refractive index and the thickness are given, can be done more easily. The present paper gives tables and graphs of transmission, reflection, and the rate of phase shift for the normal incidence and also for the oblique incidence angle 70° in the index region, which would be convenient in connection with measurements with silver films. From author's summary

3454. Feldmann, A., Nomograms for reinforced concrete beams, *Concr. Constr. Engng.* **46,** 5, 139–145, May 1951.

3455. Bosschaart, J. C., The drawing of nomograms with applications in naval architecture (in Dutch), *Schip en Werf* **17,** 22, 23; 482–487, 508–511; Oct., Nov. 1950.

Author gives short survey of theory underlying the construction of nomograms of various kinds with application to some formulas familiar to the naval architect and marine engineer.

L. Troost, Holland

3456. General Plate Division, Metals & Controls Corp., Attleboro, Mass., Corrosion resistance of typical laminated metals, *Prod. Engng.* **22,** 5, p. 211, May 1951.

3457. Wagner, K. W., Operational calculus and Laplace transformation with applications in physics and technic, 2nd ed. (**Operatorenrechnung und Laplacesche Transformation nebst Anwendungen in Physik und Technik**), Leipzig, Johann Ambrosius Barth, 1950, xviii + 471 pp. DM 42.80.

Book is a comprehensive treatment of Heaviside operational methods applied to the solution of ordinary and partial differential equations and integral equations in physical and technical problems. Wagner occupies a prominent place among those who have placed Heaviside's methods on a sound theoretical basis, but in this treatise the emphasis is on procedures useful to the engineer and technician. Examples are worked through from mechanics, heat conduction, electrotechnics, acoustics, wave propagation, and chain reactions.

Present work is a second and enlarged edition of the first one issued in 1939. Table of transforms has been enlarged. Equivalence between transforms of Laplace and Heaviside is stressed. The relation to other transforms such as those of Riemann-Liouville and Mellin is exhibited.

The table of 184 transform formulas will be useful. Work will be valuable to those using operational methods to solve boundary-value problems of physics and engineering.

H. J. Ettlinger USA

Mechanics (Dynamics, Statics, Kinematics)

(See also Revs. 3507, 3743)

3458. Guillet, G. L., and Church, A. H., Kinematics of machines, 5th ed., New York, John Wiley & Sons, Inc., 1950, vii + 297 pp. $4.

American textbooks on kinematics and mechanisms were criticized years ago for inconsistency and backwardness [*Mech. Engng.* **64,** pp. 273–278 and 744–751 (disc.), 1942]. This had no effect on producing better and more systematic texts. Present book is no exception.

Although, obviously, the book is meant for students, it should have been more systematic. It brings the reader general definitions, discusses displacement, velocity and acceleration, instant centers, velocity and acceleration in plane motion, deals with the slider-crank mechanism, cams, rolling contact, gears, gear trains, flexible connectors, and miscellaneous mechanisms.

All these subjects, with the exception of gears, are treated very superficially. As the authors deal only with rigid and flexible members, it might have been expected that the motion of a rigid system relative to another fixed rigid system would have been dealt with first. Except for the simplest problems of displacement, velocities, and accelerations, nothing of such a nature has been attempted. One looks in vain for the theory of curvature and radii of curvature and their relations, which is of prime importance to the designer, who constantly has to use radii of curvatures as links in mechanisms. The theorem of the Coriolis acceleration is cited, but is, in spite of its simplicity, not derived. On the other hand, instant centers are considered at great length, and much is made of the theorem of three centers, first discovered by Aronhold, not by Kennedy [see "Mechanics of machinery," by A. B. W. Kennedy, 1886, preface, p. vii].

In dealing with the slider-crank mechanism, inversion is explained and the four inversions of this mechanism outlined, an error having been committed, Figs. 5.12 and 5.13 being examples for the second, not third inversion.

There are other errors, such as the confusion of kinematic chains and mechanisms, as in the sentence, "These chains are known as simple mechanisms"; also, the definition of displacement: "change of position of a body with respect to a point" should read "with respect to a reference body," as rotation about a point can neither be detected nor measured.

Cams, as usually, are treated mostly from the point of view of drawing them. There is neither a clear classification nor a uniform treatment. Only the most obvious space cams are considered; globoid cams, although they have numerous applications, are omitted except for one case, Fig. 6.37, and cam levers are likewise absent.

Rolling contact should have been considered under the heading of "centrodes" or rolling curves. The geometrical construction of rolling profiles, but not their analytic determination has been dealt with. Noncircular rolling curves are not treated except for rolling ellipses, which occupy much space.

The chapter on gears is perhaps the best. A successful attempt has been made to discard irrelevant features and to lay stress on the gearing problem itself and its fundamental law. The sliding action of gear teeth has been referred to, but no relationship for the amount of sliding of the profiles at the various points has been given. Tooth interference and its consequences are dealt with, as are internal gears. Brief treatments of helical gears and worm gears are given, and hypoid gears are briefly mentioned with references being given. Gear trains are treated briefly, the treatment being simple and seemingly adequate for solving more complex gear train problems.

Flexible connectors, in spite of their importance as belt drives, etc., have been treated rather sketchily and incompletely, power drives only being considered. Rope and V-belt drives are briefly described. One looks in vain for numerous important belt and rope arrangements. Chain drives are treated superficially.

The worst is the concluding chapter on "miscellaneous" mechanisms, which contains a heterogeneous agglomeration of ratchets, straight line motions by Watt and Crosby, and by Peaucellier, and, in addition, the pantograph, elliptical trammel, Oldham coupling, Hooke's joint, and constant speed joints, none of which is dealt with properly. The large number of intermittent mechanisms and their various classes have not even been mentioned (in spite of a reference published in *Mech. Engng.* 62, pp. 537–542, 1940.)

Mechanisms containing fluid elements, though exceedingly numerous have not been mentioned nor has the kinematic synthesis of mechanisms.

References in which the student can find further information are scarce, and one (p. 110) on Mitthell's paper [*Mech. Engng.* 72, 467–471, 1950] is incorrectly given, it having been published in July and not in June. Additional references would have improved the value of the book.

In summing up, it appears that the book has considerable shortcomings and can, thus, be considered only as an introduction to machine kinematics. In spite of this, the book may be recommended, for its real advantage lies in the numerous examples given, in part, in the questions which are rather well selected (some with answers) and are appended to each chapter. Drafting-room problems are likewise of value, although problem 2 contains an error on plate 2, the points A, D, P having to lie in a straight line if proportionality of the motions of D and P are required.

A. E. Richard de Jonge, USA

[See under "Communications" in this issue.]

3459. Schnarbach, K., Cam drives and link drives for the control of processing machines (in German), *Z. Ver. dtsch. Ing.* **93,** 9, 223–228, Mar. 1951.

Relative advantages of cams and link drives are compared. Practical selection of link drives is illustrated on a packaging machine requiring several synchronized movements with prescribed rest periods. G. Sved, Australia

3460. Nadile, A., On the motion around a fixed point of a heavy rigid body with center of gravity located on one of the principal planes of inertia (in Italian), *Matematiche, Catania* 5, 68–82, 1950.

Author finds two types of uniform rotation in the problem of the motion of a rigid heavy body about a fixed point with center of gravity located on one of the principal planes of inertia relative to the fixed point. Certain motions involving only quadratures and the integration of a Riccatti equation are also found. These latter depend on five constants of integration. D. C. Lewis, USA

3461. Zeuli, T., On the motion of a heavy asymmetric rigid body with center of gravity located on the axis of a circular section of the inertia ellipsoid (in Italian), *Univ. Politecn. Torino R.C. Sem. Mat.* 9, 263–295, 1950.

Author treats the problem described in the title by expanding the unknown functions in powers of a parameter, the coefficients being functions of the time. The parameter is such that its vanishing reduces the problem to the classical case of the symmetrical gyroscope. D. C. Lewis, USA

3462. Griseri, B., Meromorphic solutions of the equations of motion around a fixed point of a heavy rigid body with center of gravity located on the axis of a circular section of the inertia ellipsoid relative to the fixed point (in Italian), *Univ. Politecn. Torino R.C. Sem. Mat.* 9, 225–243, 1950.

Solutions of the problem described in the title which are meromorphic in the plane of the complex variable t with a pole at the origin are studied by the author. Two such solutions always exist and a third is also possible, provided the constants in the original equations satisfy a certain condition.

D. C. Lewis, USA

3463. Manacorda, T., On the motion of a heavy rigid body around a fixed point (in Italian), *Riv. Mat. Univ. Parma* 1, 383–391, 1950.

The equations of motion of a heavy rigid body with a fixed point are written down on the assumption that the center of gravity lies on one of the principal planes of inertia relative to the fixed point. Author then finds two invariant relations in the form of expressions for the components of angular velocity as power

series in a certain linear combination of these components. He thus arrives at a representation for the general solution and exhibits how a special case of a partial solution by quadratures (effected by Hess) fits into the theory. D. C. Lewis, USA

3464. Emersleben, O., The maximum throwing range on horizontal planes and on concentric spherical surfaces (in German), *Technik* **5**, 12, 601–606, Dec. 1950.

A mathematical investigation is given for the maximum throwing range, on horizontal planes of different altitudes and on concentric sphericals, of a particle acted on only by gravitation; both parabolic and elliptic (Keppler ellipsis) trajectories are contemplated. Author concludes that the maximum range is obtained when the initial velocity is the bisector of the vertical direction and the line drawn from the origin to the terminal point of the trajectory. Aurel A. Beleş, Rumania

3465. Hrones, J. A., and Nelson, G. L., Analysis of the four-bar linkage. Its application to the synthesis of mechanisms, Mass. Inst. Technol., Tech. Press; New York, John Wiley & Sons, Inc., 1951, xx + 730 pp. $15.

The beginning of a stated broader program of publishing information for more effectively designing pinned linkages, which often have advantages over cam actions. Four-bar linkages are surveyed, being classified according to whether (*a*) one crank is rotary through 360° and the other oscillatory; (*b*) both cranks rotary; and (*c*) both cranks oscillatory. Criteria for the classes are given in terms of linkage ratios and exhibited in a comprehensive classification chart. Included are a summary of instructions and contents, and a brief analysis of the four-bar linkage, with illustrations showing how the extensive graphical information in the subsequent layouts may be applied to synthesizing linkages in general.

Main substance of the work consists of an ordered arrangement of many kinematic layouts of crank-and-rocker linkages in class (*a*), the ratios of link lengths being varied by finite steps over a useful range. All layouts deal with a selected field of points on the connecting link, one fifth of the field being treated in each layout, a set of five layouts covering a given link ratio, and one of each set dealing with some points on the rocker. The sets form groups in which two of the basic link ratios are held constant while the third is progressively varied. Each layout includes trajectories of points in that part of the field assigned to it.

Over 7000 mechanically plotted closed trajectories appear, each composed of 72 variable-length dashes, each dash punctuated by an end dot, the distances between dots measuring displacements of the field point in question for successive 5° crank displacements. Counting of the dashes is tantamount to measuring crank angle; the distances between adjacent dots, when divided by time, as determined by the driving crank when moving at constant velocity, measure the average velocity along the trajectory between such dots. By vectorially substracting velocities as represented by distances between adjacent pairs of dots and again dividing by time, an approximate measure is obtained of the total acceleration at the common dot of the adjacent pair. Thus over 500,000 solutions are quickly available, encompassing the complete displacement, velocity, and acceleration performances in the range of linkages treated.

The trajectory shapes and the displacements are quite accurate; the derived velocities are averages over short increments of displacement; the derived accelerations in many instances constitute rough approximations which are intended to be more in the nature of guides to performance and are best at the higher values. The essential idea of the work is that by scanning the many but ordered layouts, a likely trajectory may be selected (several may come close to solving a problem) and the usefulness of its tracer point studied as a combining coupler with an additional linkage, the whole to provide the desired motion. With a minimum amount of analytical work an idea may be obtained of the displacements, velocities, and accelerations involved. It is demonstrated how transparent overlays carrying layouts of desired actions may be matched to the trajectories to facilitate linkage selections, and it is suggested that interpolations may be accomplished by kinematic models.

The problem of synthesis of mechanisms by purely analytical methods is ordinarily intractable. Reviewer believes that this compendium, which simplifies the analytical work, will be of substantial value to the experienced designer.

Lloyd R. Koenig, USA

3466. Sakharnikov, N. A., On the conditions of existence of a center and focus (in Russian), *Prikl. Mat. Mekh.* **14**, 5, 513–526, Sept.-Oct. 1950.

Author treats the system of equations, $dx/dt = y + f(x, y)$, $dy/dt = -x + g(x, y)$, where f and g have power-series expansions in x and y in the neighborhood of the origin, and considers the problem of determining when the origin is a center or a focus. The results are obtained by considering the related partial differential equation $(y + f)\partial F/\partial x + (g - y)\partial F/\partial y = 0$.

R. Bellman, USA

3467. Baranov, G. G., On the solution of some problems of Chebíshev (in Russian), *Akad. Nauk SSSR Trudí Sem. Teorii Mash. Mekh.* **5**, 20, 78–107, 1948.

In order to popularize Chebíshev's ideas, author presents solutions of four guidance problems in which the application of Chebíshev's criterion of best approximation calls only for elementary analytic geometry. This criterion requires, for the best approximating curve, a maximum number of oscillations, all of the same amplitude. In this paper's problems the approximating curves can be fully analyzed for the most general case by elementary means, and then specialized to meet the conditions. The problems are: (1) Rectilinear guidance; "lambda" four-hinge linkage ($OABC$, OC fixed, $BA = BC \neq BM$, M tracing point on AB). As limit cases, third- and fifth-order contact with a straight line are obtained. (2) Rectilinear guidance; turning-block linkage OAC, OC fixed, OA constant, A-hinge, C-slider crank; tracing point M on AC. (3) Uniform straight-line motion; turning-block linkage (2) with $\tan OCA$ as the generated function of time. (4) Uniform straight-line motion; Oldham's linkage: two fixed slider cranks O and C guiding the arms of a rigid right angle BAB'. The distance of B from OC is the generated linear function. In each problem the length of the approximation stretch and the magnitude of the deviation are carefully computed.

A. W. Wundheiler, USA

3468. Cherkudinov, S. A., Design of some mechanisms approximating uniform motion (in Russian), *Akad. Nauk SSSR Trudí Sem. Teorii Mash. Mekh.* **6**, 21, 5–26, 1949.

In the original Chebíshev method the approximating function is given explicitly as a linear combination of an appropriate sequence of functions. Author proposed a modification of the method in which the approximating function is defined as an implicit function of the basic independent variable z, by means of the equation $F(v, z) = 0$, F being a polynomial in z. If v is a constant, Chebíshev's criterion requires a maximum number of oscillations at constant amplitude for the best approximation (the author does not discuss the case of variable v). The procedure was described in previous papers [*Izv. Akad. Nauk SSSR Otd. Tekh. Nauk* 1948, 1517–1530; same *Trudí* **5**, no. 20, 34–77, 1948].

Here it is applied to generation of nearly uniform motion by means of the turning-block linkage, OAB (OB fixed, OA is the crank, BA slides through A) with the following generated functions: (1) $v = \tan ABO$, (2) $v = \sin ABO$, and also to the motion of the slider of the slider-crank linkage with offset. The maximum deviations and range of approximations are computed.

A. W. Wundheiler, USA

3469. **Cherkudinov, S. A., On the design of slider-crank mechanisms generating approximately uniform motion** (in Russian), *Akad. Nauk SSSR Trudí Sem. Teorii Mash. Mekh.* **5**, 18, 5–23, 1948.

This paper further exploits the idea of the preceding one: If the point B of the coupler A of a turning-block linkage moves on a perpendicular γ to the frame OT, its motion is almost uniform. The author hence applies Chebíshev's method to solve the problem of rectilinear guidance for the turning-block linkage. If now B is connected with γ by means of a slider crank, and the turning block is suppressed, a slider-crank linkage is obtained with almost uniform slider motion. The dimensioning (in terms of the lengths of crank and slider travel and the velocity error) is discussed in detail. It is shown that the linkage crank cannot be a revolving one.

A. W. Wundheiler, USA

3470. **Cherkudinov, S. A., On the design of four-hinge linkages generating approximately uniform motion** (in Russian), *Akad. Nauk SSSR Trudí Sem. Teorii Mash. Mekh.* **3**, 9, 60–73, 1947.

A point B of the coupler rod AT of a turning-block linkage (OAT, OT fixed, OA-crank, A-pivot, T-slider crank) can be made to trace a near circle. [Of a few pertinent references, we shall mention only the most recent one: Z. Š. Blokh, see Rev. 3473 in this issue.] The author observes that if the center C of this circle is on the frame bar OT, the motion of M will be uniform. The four-hinge linkage $OABC$ with OC fixed will then yield approximately uniform circular motion of C if OA rotates uniformly. The Blokh solution is used for the dimensioning of this four-hinger in terms of the transmission ratio i, which must be between 0.5 and 2. Either both cranks revolve, or they both oscillate. A linkage is determined for which C has instantaneously zero accelerations of the first and second orders. The locus of the corresponding positions of A for a given i is a circle symmetric about OT.

A. W. Wundheiler, USA

3471. **Levitskiĭ, N. I., Mechanisms design according to Chebíshev [Sintez mekhanizmov po Chebíshevu]**, Moscow-Leningrad, Izdat. Akad. Nauk SSSR, 1949, 180 pp.

This is a leisurely, and perhaps a little wordy, account of Chebíshev's method of best approximation, and its detailed applications to some fully specialized problems. For general mathematical developments the author directs the reader to the Bernstein book on polynomial approximations ["Extremal properties of polynomials and best approximation . . .," ONTI, Moscow-Leningrad, 1937]. After a brief historical introduction, the definition of Chebíshev's best approximations are given; the basic theorem is stated without proof; the Chebíshev polynomials are determined and elaborately tabulated; another example of a Chebíshev system presented; and the problem of a monotone best approximation to zero solved for a polynomial. Turning now to mechanisms, the choice of definition of deviation is discussed: weighted differences; $F(x(t), y(t))$ where $F(x, y) = 0$, is the approximated curve and $x(t)$, $y(t)$ the approximating one; and simplifying truncation of deviations. Polemical remarks are directed at Z. Š. Blokh's favoritism for the $F(x(t), y(t))$ deviation [this criticism is justly voiced by many Russian authors, without apparent effect]. The second part of the book (96 pp.) devotes 60 pages to Chebíshev's favorite simple and strikingly fertile mechanism: It is a four-bar linkage $OABC$, O and C fixed, $AB = BC \neq BM$, where M is the tracing point (of the coupler BC), the angle $ABM = \omega$ being constant. There are three parameters $OA/AB = r$, $OC/AB = d$, and ω. For $\omega = \pi$ the mechanism is called a "lambda linkage." The Cartesian equation of M's trajectory is derived, and a general survey of its possible types (in terms of cusps, inflections, double points) is made. The method leads to a number of algebraic equations for the parameters, positions of maximum deviation, and the magnitude of the deviation. The dimensions r, d, ω are determined for the following curves approximated by M (mostly with weighting): (1) straight-line segment; (2) two straight-line segments; (3) circular arc; (4) two circular arcs; (5) hyperbolic arc (with several weighting assumptions). Reduction of the number of oscillations (for reasons of reality) must be sometimes used. A slider-crank linkage $OBCM$ with offset is taken up next: OB is the crank, BC the connecting rod, C the slider. The angle $BCM = \omega$ is constant and M is the tracing point. The dimensions are determined for an approximately straight-line segment for M. Finally, a lambda and a turning-block mechanism are determined for uniform rectilinear motion.

A. W. Wundheiler, USA

3472. **Dimentberg, F. M., A general method for finite displacements of spatial mechanisms and on certain passive constraints** (in Russian), *Akad. Nauk SSSR Trudí Sem. Teorii Mash. Mekh.* **5**, 17, 5–39, 1948.

This may be the first application of Clifford's dual numbers to the position problem of four- to seven-bar space linkages, and the power of this method is here most impressive. The theory of screws in dual-number symbolics is briefly but adequately presented: If $\mathbf{u}$ is a sliding vector, $(a + \omega b)\mathbf{u}$ is a screw, $a\mathbf{u}$ being the translation, $b\mathbf{u}$ the angular velocity, and ω a complex unit with $\omega^2 = 0$. The dual angle α of two sliding unit vectors $\mathbf{u}$, $\mathbf{v}$ is $\alpha_0 + \omega\alpha_1$, where α_0 is the angle and α_1 their shortest distance. Their scalar product $\mathbf{u}\cdot\mathbf{v} = \cos\alpha = \cos\alpha_0 - \cos\alpha_1 \sin\alpha_0$, and for their vector product $|\mathbf{u} \times \mathbf{v}| = \sin\alpha = \sin\alpha_0 + \omega\alpha_1 \cos\alpha_0$, its axis being the common normal of $\mathbf{u}$ and $\mathbf{v}$. With these definitions the formulas for screw displacements become identical with those for pure finite rotations if all vectors are replaced by sliding vectors, and all angles of rotation φ_0 by corresponding dual angles of displacement $\varphi = \varphi_0 + \omega\varphi_1$ where φ_1 is the length of the translation. A basic formula defines the sliding vector u_1' obtained by applying the screw motion $\varphi\mathbf{u}$ to a sliding vector $\mathbf{u}_1$:

$$(1 + \phi^2)u_1' = (1 - \phi^2)\mathbf{u}_1 + 2(\mathbf{u}\cdot\mathbf{u}_1)\mathbf{u}\phi^2 + 2(\mathbf{u} \times \mathbf{u}_1)\phi,$$

where $\phi = \tan\varphi/2 = \tan\varphi_0/2 + \omega\varphi_1/2(1 + \tan\varphi_0/2)$.

A four-bar spatial linkage (one turning pair and three cylindrical pairs) is defined by the four dual angles between adjacent axes. The bars are assumed as the common normals of adjacent axes. One of the bars forming the turning pair is the crank, and its dual angle $\varphi = \varphi_0 + \omega\varphi_1$ (φ_1 is constant here) is the independent variable in terms of which the other dual angles between adjacent bars are desired. The method consists in expressing the closure of the chain: Imagine the bar 12 subjected to the screw displacement which makes it align itself with 14 in the position $12'$($2'$ is the known new position of axis 2); similarly, let $43'$ be the position of 43 after a screw displacement $\psi\mathbf{u}_4$ to assignment with 41. Now express that the axis 2 obtained after the screw displacement $\varphi\mathbf{u}_1$ from $2'$ and the axis 3 obtained after the screw displacement $\psi\mathbf{u}_4$ from $3'$ make the dual angle defined by the linkage dimensions. Considerable algebra is involved, but explicit formulas are obtained for $\tan\psi_0/2$ and ψ_1 in terms of $\tan\varphi_0/2$ and φ_1. Similar formulas are obtained for the other dual angles. A discussion is made of singular cases: the translational part of the

displacement at one cylindrical hinge may be either indeterminate or zero. In the latter case the cylindrical hinge becomes a turning pair. All such linkages (of which Bennett's mechanism is a classical example) are determined by means of the formulas previously mentioned. Four-bar linkages with turning and spherical pairs are considered briefly, and a relation between dual angles (screw displacements) at adjacent axes derived. The same problem is solved for all possible five-bar linkages (again after considerable algebraic labor). There is only one type of a six-bar or a seven-bar linkage; only brief indications of the manner of application of the method are given. The paper ends with a brief rederivation of the spatial geometric construction of the resultant of two screw displacements modeled on the corresponding construction for finite rotations. This is applied to the derivation of the screw displacement of the link 34 of a seven-bar mechanism if the rotations at the hinges 1, 2, 3 are known (all the pairs are turning pairs in this case).

A. W. Wundheiler, USA

3473. Blokh, Z. Š., Approximate mechanisms design [Priblizhenīĭ sintez mekhanizmov], Moscow, Gosud. Nauchno-Tekhn. Izdat. Mashinostroitel'noĭ Lit., 1948, 171 pp.

In comparison with the subject of preceding Rev. 3471, this is a much more concentrated coverage of a larger assortment of problems solved by Chebíshev's methods. General questions of this method are treated in the first 15 pages, and linkage geometry (Roberts' theorem, trajectory equations $F(x, y) = 0$) in the following 14 pages. The remainder represents an album of specialized problems. In each there is involved: a mechanism with some variable dimensions; a point of the mechanism (sometimes with one variable coordinate); and a curve (or motion) subject to best approximation. The solution consists in: choosing the measure of the deviation so that it can be represented as a linear combination of the elements of a Chebíshev system (the author ignores the criticism of $F(x, y)$ as deviation measure); finding the coefficients of the best approximation; finding the corresponding dimensions of the mechanism, the maximum deviation, the length of the approximating segment, and the range of crank motion (this is frequently laborious, and occasionally series expansions must be substituted for the exact functions). The transmission angle (between the coupler and the driven member) is sometimes computed. The best approximation may be not real, and then a poorer one (with fewer oscillations) must be chosen. Considerable technique and drudgery are involved in some problems. Many specific results were given by Chebíshev without proofs; they are supplied here.

Only four mechanism types are really considered: (A) the symmetric four-bar linkage C_1ABC_2, C_1 and C_2 fixed, $AC_1 = BC_2$, with symmetric position of the tracing point M ($MA = MB$ constant); (B) Chebíshev's asymmetric linkage: C_1ABC_2, C_1 and C_2 fixed, $C_1A = AB = BM$ where M is the tracing point; (B′) the lambda mechanism (a special case of (B) with $\angle BAM = \pi$); (C) slider crank (with offset), M on connecting rod; (D) turning block (connecting rod is element of a sliding pair). Transformations by means of Roberts' theorem into four-bar and six-bar linkages are also noted.

The problems are arranged according to type of guidance and deviation measure. First, straight-line tracing is solved for (A) and (B): Slit tracing (a narrow loop very close to a straight line) is solved for (A), (B), and (B′). Using $y^2 - k^2x^2$ as deviation (rather than $y - kx$) a simpler solution is obtained for (B′) or (A) with M at the center of the connecting rod (Evans' linkage). Using $F(x, y)$ as the deviation measure, where $F(x, y) = 0$ is the stylus curve, the Watt mechanism is obtained. For (C) ,the stylus is on the connecting rod and the deviations considered are $x - x_0$, $F(x, y)$. For (D) the stylus is on the connecting rod, and the deviations chosen are $x - x_0$.

Second, tracing of a circle $T(x, y) = 0$ is solved for (B), the deviation being $T(x, y)$ and $F(x, y)$ (leading to Watt's mechanism). The case when the whole trajectory is close to a circle is singled out. For (D) and (C), $T(x, y)$ is used as deviation measure. Tracing of a parabola is solved for (C) and $F(x, y)$ as deviation. For the curve $(x^2 + y^2)^2 + k(x^2 + y^2) + mx + n = 0$ the left-hand side is used as deviation and (D) as the mechanism.

Third, (C) with an offset is designed for uniform motion of the slider and a general four-bar linkage for uniform rotation of the lever. In both cases, certain polynomials in time and velocity (resulting from angle elimination) are used as deviation measures (an obviously questionable procedure). (D) is designed for uniform rotation of the lever, and for uniform "vertical" velocity of a point of the lever. (C) is designed for harmonic motion of the slider (a polynomial in the sine of the crank angle is used as the deviation).

The last chapter treats an alternative procedure of series development off the main stream of the best-approximation method. It can be applied when the crank angle varies but little. In the design for high-order tangency the following ingenious device of Chebíshev is useful: Two bars KN and LM, pivoted at N, $KN = LN = MN$, are known as Chebíshev's dyad. If M is fixed, and L moves in a λth-order contact with a straight line, M will move in a $(2\lambda + 1)$th-order contact with a straight line. Three six-bar linkages are designed by the series-expansion method yielding, respectively, 7th, 8th, and 6th-order contact. One four-bar linkage yields 6th-order contact with a tangent parallel to the base. A lambda mechanism yields a 5th-order contact (and a linkage spectacularly superior to Evans'). The last two topics are: the best choice of circular tooth profile; and the dimensioning of a centrifugal governor for maximum sensitivity. The problem is reduced to the series method; Prelle's and Chebíshev's designs are obtained.

A. W. Wundheiler, USA

Gyroscopics, Governors, Servos

3474. Enskog, G., Modern regulation of turbines (in Swedish), *Tekn. Tidskr.* **81**, 14, 269–275, Apr. 1951.

For water turbines, two different systems of regulators are used: mechanical and electrical. Using mathematical methods to study the behavior of these regulators, author concludes that the equation system for an electrical regulator is identical with that for a mechanical. Next there is a brief description of the Swedish Kanova regulator and some references to literature.

A. R. Holm, Denmark

3475. Jones, A. L., and Briggs, B. R., A survey of stability analysis techniques for automatically controlled aircraft, *Nat. adv. Comm. Aero. tech. Note* 2275, 112 pp., Jan. 1951.

Report is a modern survey of a field considered by reviewer in 1939 in *Nat. adv. Comm. Aero. tech. Note* 700. Authors have collected the general principles of servomechanism design which are now common practice and describe their application to the automatic control of airplanes. Emphasis is placed on the stability of the system as shown by absence of divergence and upon the criteria of speed of response, steady-state error, time to damp, and amount of initial overshoot. Practical numerical examples show applications of the methods of a typical aircraft.

While the work represents a valuable introduction to the subject, the present state of the art is by no means covered. The work of Kochenberger on the analysis of high-order dynamic sys-

tems controlled by "on-off" servomechanisms is not mentioned, nor is the thorough, practical, and extremely valuable work by Millikan and associates at the Cornell Aeronautical Laboratories. Recent work, perhaps published too late to be included, is that reported by J. B. Rea.

Although one of Wiener's books is given as a reference, no application is made of the Wiener theories. It appears to reviewer that the problem of "best" automatic control of aircraft is one of optimizing the average value of some parameter over a period of time. This may be "passenger comfort" as measured by rms acceleration in the case of aircraft intended for commercial use, deviation from a straight line in the case of bombing aircraft, or miss distance from a target in the case of a guided missile. The development of techniques for servo analysis by statistical methods now makes such objectives seem well within reach. A survey of applicable statistical methods might be a valuable addition to subject report. Herbert K. Weiss, USA

3476. Guarnaschelli, F., Methods for the study of servomechanisms (in Italian), *Aerotecnica* **30,** 6, 316–322, Dec. 1950.

Brief review of well-known analytical, operational, and graphical methods of servomechanism analysis and synthesis.
William A. Mersman, USA

3477. Miller, K. S., and Schwarz, R. J., Analysis of a sampling servo mechanism, *J. appl. Phys.* **21,** 4, 290–294, Apr. 1950.

Authors treat a servo loop that samples the servo errors periodically and "clamps" these samples in the interim. The solution is found by first solving the open loop with the help of the one-sided Green's function, and matching boundary conditions at the sampling instants. Stability relations similar to those given by MacColl ["Fundamental theory of servomechanisms," New York, Van Nostrand, 1945] are found. Unfortunately, the authors have used the symbol $\epsilon(t)$ for two distinct functions, namely, the error signal before sampling, and the output of the clamping mechanism.
Courtesy of Mathematical Reviews G. R. Stibitz, USA

Vibrations, Balancing

(See also Rev. 3573)

3478. Traill-Nash, R. W., The symmetric vibrations of aircraft, *Aero. Quart.* **3,** part 1, 1–22, May 1951.

Author derives the frequency equation for a complete aircraft regarded as free-free system. Using the approximation of "lumped mass," the system is divided into four branches. Equations in matrix form are set up which define the elastic and absolute displacements of each branch. With these submatrixes, general frequency equations are obtained from which the eigenvalues may be calculated.

The above analysis is carried out for wings of conventional type and a procedure is indicated for the treatment of swept wings. Author points out similarity as well as difference in approach to the same problem as solved by J. Morris and G. S. Green.

Reviewer feels that a comparison between calculated frequencies and those observed experimentally would be of interest. J. Morris and G. S. Green carried out experimental investigations and found that frequencies obtained by calculation were in good agreement with experiments up to fifth overtone, while modes were in good agreement up to second overtone.
Wilhelm Ornstein, USA

3479. Morris, J., An application of the escalator process, *Aircr. Engng.* **23,** 267, 136–137, May 1951.

The escalator process, developed by Morris and Head for the solution of vibration problems, is shown to be applicable to certain unusual species of Lagrangian-type frequency equations. It is indicated that these forms arise in such problems as the whirling of contrapropeller systems and the mechanical vibration of an aircraft. Discussion is purely mathematical. Reviewer believes value of paper would have been enhanced if some insight had been provided into nature of physical problems yielding the equations discussed. Gabriel Isakson, USA

3480. Seki, T., On the approximate estimation of the natural torsional frequency of the Diesel engine crankshaft, *Memo. Fac. Sci. Engng. Waseda Univ. Tokyo,* no. 14, 24–25, 1950.

Engine inertias are lumped and connected to the driven inertia by a shaft from Carter's formula. Formula may be useful to get rough trial for the Holzer table. A. C. Hagg, USA

3481. Takahasi, T., On a quasi-harmonic vibration of a flexible strip rotating with variable speed, *Memo. Fac. Sci. Engng. Waseda Univ. Tokyo,* no. 14, 28–29, 1950.

Mathematical exercise on a rotating strip under centrifugal force only. Period and amplitude decreases or increases with acceleration or deceleration, respectively. Experiments check the theory qualitatively. A. C. Hagg, USA

Wave Motion, Impact

(See also Revs. 3539, 3548, 3598, 3636, 3738)

3482. Broer, L. J. F., On the propagation of energy in linear conservative waves, *Appl. sci. Res.* **A2,** 5–6, 329–344, 1951.

Direct application is made of method of stationary phase [Eckart, C., *Rev. mod. Phys.* **20,** 2, p. 399, 1948] to show that, for conservative wave motion, the rate of energy propagation equals the group velocity. An observation is made that care must be taken to define the energy density in such a way that the average energy density equals the ratio of rate of work done to group velocity.
Keeve M. Siegel, USA

3483. Satô, Y., Rayleigh waves propagated along the plane surface of horizontally isotropic and vertically aeolotropic elastic body, *Bull. Earthq. Res. Inst. Tokyo Univ.* **28,** parts 1–2, 23–29, Jan.-June 1950.

Author proves mathematically that one and only one sort of Rayleigh waves exists in horizontally isotropic but vertically aeolotropic medium. Starting from equations of motion in elastic medium stated above, author derives characteristic equation for Rayleigh-type waves. On examining the solutions of the equation, existence of one and only one sort of Rayleigh waves was ingeniously proved. Hence, we know only one kind of Rayleigh waves should be found on actual seismograms, even in the case of the said anisotropy in the earth's crust. Some misprints are found in the expressions (2.1), (2.3), (2.4), and (2.8).
Tsuneji Rikitake, Japan

3484. Cooper, R. I. B., and Longuet-Higgins, M. S., An experimental study of the pressure variations in standing water waves, *Proc. roy. Soc. Lond. Ser. A,* **206,** 1086, 424–435, May 1951.

Paper deals with experimental measurement of second-order pressure variations for cases of complete and partial standing water waves and utilizes such pressure measurements to determine the reflection coefficients of sloping beaches. Such second-order

pressure variations in ocean waves may be one cause of microseisms. [See AMR **4**, Rev. 2293.] Waves of 0.45 to 0.50-sec period were generated in a tank 970 cm long by 50 cm deep by 24 cm wide and associated pressures measured with a quartz crystal hydrophone. Complete standing waves were produced by total reflection of generated progressive waves from a vertical bulkhead. Partial standing waves were produced by partial reflection of generated waves by a vertical barrier extending various distances below the surface.

Good agreement with theory was obtained for measurements of complete standing waves. The second-order pressure variation was found to be proportional to wave height squared, of twice the generated wave frequency, and independent of depth or horizontal position in the wave system. Reviewer believes part of the work concerned with partial standing waves needs further study. Authors state second-order pressure variations for partial standing waves to be proportional to product of incident and reflected wave heights, hence proportional to βa^2, where β is reflection coefficient and a is incident wave amplitude. Reviewer suggests that second-order term for this case should be proportional to $\beta^2 a^2$. The experimental work is not conclusive on this point since it hinges on theoretical reflection coefficients of partially submerged vertical barriers, which have not been confirmed by independent experiments. Measurements of beach-reflection coefficients suffer from the same ambiguity.

John H. Carr, USA

3485. Hughes, D. S., and Jones, H. J., Elastic wave velocities in sedimentary rocks, *Trans. Amer. geophys. Un.* **32**, 2, 173–178, Apr. 1951.

By piezoelectric pulse method, dilatational and rotational wave velocities in sedimentary rocks are measured directly under high pressure and temperature. Velocity increase with pressure and allied hysteresis phenomena are disclosed. Importance of past treatment on elastic properties of rocks is also suggested. Wave velocities and elastic properties are computed as functions of depth, assuming a temperature gradient of 9.5 C per 1000 ft. Corresponding to 35,000 psi, the maximum depth amounts to about 50,000 ft. At pressure above 15,000 psi, velocity change becomes small. On comparing the velocities to those obtained by seismic surveys, the laboratory values seem to be slightly lower. Though authors suggest a possible dispersion effect, no detailed discussion is attempted. Tsuneji Rikitake, Japan

3486. Homma, S., On the propagation of dispersive seismic waves, *Geophys. Mag. Tokyo* **22**, 3, 209–236, Feb. 1951.

Author discusses propagation of dispersive surface waves in considerable detail. Method of stationary phase is used to evaluate order of magnitude of maximum amplitude of wave at large distances. It is shown that if the group velocity has a maximum or minimum value for a finite frequency, the maximum amplitude is $O(r^{-5/6})$. This agrees with the results of Pekeris [*U. S. Geol. Soc.*, Memoir 27, 1948].

Author further discusses, not very adequately, the cases when the maximum or minimum group velocity coincides with zero or infinite frequency. Finally, the problem of an absorptive medium is discussed. Author concludes that the maximum amplitude is $O(r^{-1})$, i.e., smaller than in an Airy phase.

J. M. Jackson, Scotland

3487. Lee, E. H., and Wolf, H., Plastic wave propagation effects in high speed testing, *Grad. Div. appl. Math. Brown Univ. Tech. Rep.* A11-48, 33 pp., 1950.

Authors show that neglect of plastic wave-propagation effects may invalidate stress-strain measurements in high-speed material testing, due to the presence of nonuniform strain variations over the specimen as the testing speed is increased. A particular test arrangement is analyzed in detail, and it is shown that the application of one-dimensional plastic wave theory accounts for earlier discrepancies between experimental results and theoretical predictions in that case.

Courtesy of Mathematical Reviews F. B. Hildebrand, USA

3488. Keulegan, G. H., Wind tides in small closed channels, *J. Res. nat. Bur. Stands.* **46**, 5, 358–381, May 1951.

Author has carried out a systematic laboratory investigation concerning the various factors that may affect the stress of the wind on a water surface. Mean values of stress are obtained from the difference in water level on the upwind and downwind sides. The analysis of the data is more consistent if it could be supposed that the stress is made of two parts: the first being due to skin friction (without waves), and the latter due to the form resistance of the wave crests. Separation of these effects is made possible by the discovery that the addition of detergent would inhibit the formation of waves. The skin friction obeys the ordinary theory of smooth plate resistance. The resistance due to waves is proportional to the square of the excess of the wind speed above some critical velocity, equal approximately to the minimum velocity for wave generation. This critical velocity depends on the water depth for the relatively shallow depths for which the experiments are performed. Walter H. Munk, USA

Elasticity Theory

(See also Revs. 3498, 3510, 3511, 3569)

3489. Segedin, C. M., Note on a penny-shaped crack under shear, *Proc. Camb. phil. Soc.* **47**, part 2, 396–400, Apr. 1951.

Author considers problem of penny-shaped crack (plane crack bounded by a circle) in infinite medium where two faces of the crack are subjected to constant shearing stress and other components of stress across it are vanishing. Stress in penny-shaped cracks in an infinite medium under tension normal to the face has been considered previously. Problem is found to reduce to same boundary problem as penny-shaped crack under normal tension. An expression is also found for change in strain energy which results when crack forms in the uniformly sheared material.

J. S. Aronofsky, USA

3490. Rivlin, R. S., and Saunders, D. W., Large elastic deformations of isotropic materials. VII. Experiments on the deformation of rubber, *Trans. roy. Soc. Lond. Ser. A*, **243**, 865, 251–288, Apr. 1951.

In earlier parts, Rivlin has discussed theoretically the deformation of ideal highly elastic, incompressible, isotropic materials. This part deals with the application of this theory to the deformation of vulcanized-rubber sheets and cylinders. It is assumed that the stored-energy function of the ideal material may be written as $W = C_{10}(I_1 - 3) + C_{01}(I_2 - 3)$, where I_1 and I_2 are strain invariants. $I_1 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2$ and $I_2 = 1/\lambda_1^2 + 1/\lambda_2^2 + 1/\lambda_3^2$. λ_1, λ_2, λ_3 are the principal extension ratios and are defined as $\lambda_1 = 1 + e_1$, $\lambda_2 = 1 + e_2$ and $\lambda_3 = 1 + e_3$. The applied forces and stresses necessary to produce certain deformations can then be calculated as functions of $\partial W/\partial I_1$ and $\partial W/\partial I_2$.

Experiments on vulcanized-rubber sheets and cylinders subjected to systems of simple and combined stresses and strains are described. Experimental results are used to check the theoretical calculated forces necessary to produce specified deformations. Experimental results are also used to check assumed dependence of W on I_1 and I_2. Experimental conditions were such that effects

of hysteresis, internal friction, and permanent set were minimized. Correlation between calculated theoretical values and experimental results is reasonably good. Yoh-Han Pao, USA

3491. **Conforto, F., New development in the numerical solution of partial differential equations of higher technology** (in German), *Cons. Naz. Ricer. Publ. Ist. appl. Calc.* no. 291, 4 pp., 1950.

Problem under consideration is that of finding the displacement vector field in a three-dimensional elastic body when the displacements are given on part of its surface, and the surface tensions on the remainder. Use is made of a method by M. Picone which reduces this boundary-value problem to the solution of an infinite system of linear algebraic equations. For computational purposes, the system is broken off at a finite number of equations. It is shown that $3(n+1)^2$ equations are necessary to obtain an nth-order approximation to the exact solution. Use of automatic computing machinery is suggested for practical problems of this type which may be encountered in the analysis of dams.

R. Drenick, USA

3492. **Shield, R. T., Notes on problems in hexagonal aeolotropic materials,** *Proc. Cambr. phil. Soc.* **47,** part 2, 401–409, Apr. 1951.

Three-dimensional stress distributions in bodies having hexagonal aeolotropic elastic symmetry were considered by H. A. Elliott [AMR 2, Rev. 166; 4, Rev. 112]. Present author reduces problem to determination of two harmonic functions. He uses this theory to extend certain known results for isotropic bodies. He considers first a thin infinite plate bounded by planes $z = \pm h$, loaded by the line load on z-axis parallel to x-axis. Author starts with known stresses in infinite solid in plane strain under line load on z-axis parallel to x-axis. A second stress system is then written down which, when added to first system, frees the planes $z = \pm h$ from stress. Also considered is an infinite solid under uniform tension at infinity, with elliptical crack normal to tension; also case when rigid elliptical stamp is forced into a semi-infinite solid; and case when a point load acts on a semi-infinite solid at an interior point normal to plane face. G. E. Hay, USA

3493. **Synge, J. L., Upper and lower bounds for the solutions of problems in elasticity,** *Proc. roy. Irish Acad. Sec. A*, **53,** 41–64, 1950.

The problem of finding numerical upper and lower bounds for a real valued function u at a point P (the function u being a priori unknown, but about which some information is known; e.g., u is the solution of a boundary-value problem, so that a partial differential equation and boundary conditions satisfied by u are known) consists in finding real numbers $A(P)$ and $E(P)$, the approximation and the error at P, respectively, such that $|u(P) - A(P)| \leq E(P)$, where the numbers $A(P)$ and $E(P)$ may be computed explicitly in terms of known functions which are chosen, making use of the information known about u. A solution of this pointwise bound problem, for interior points, for the solution (i.e., u being any displacement) of the first boundary problem (surface displacements given on the boundary) of three-dimensional, homogeneous, isotropic elasticity, was given by the reviewer and H. J. Greenberg [AMR 1, Rev. 306]. The present paper gives a solution of the pointwise bound problem, also for three-dimensional homogeneous isotropic elasticity. In the first boundary-value problem, surface displacement being given, bounds are obtained for the dilatation and the displacement. In the second boundary-value problem, surface stress being given, bounds are obtained for the sum of the principal stresses and for the stress components. The argument depends essentially on the mean value theorem for harmonic functions. All the inequalities given reduce to equalities when the domain occupied by the elastic body is a sphere, and several formulas for the various elastic quantities at the center of the sphere are deduced from this.

Courtesy of Mathematical Reviews J. B. Diaz, USA

3494. **Mindlin, R. D., and Cheng, D. H., Thermoelastic stress in the semi-infinite solid,** *J. appl. Phys.* **21,** 9, 931–933, Sept. 1950.

The solution for a center of dilatation [AMR 4, Rev. 1474] in the semi-infinite solid is applied to the case of an expanding spherical inclusion in an elastic half space. Also, it is shown that for the semi-infinite body with stress-free surface the potential problem is the same as for the infinite body. The plane boundary is freed of stress by applying a center of dilatation, a double force, and a doublet, each of proper strength, at the image point of each center of dilatation in the body. D. C. Drucker, USA

3495. **Seugling, W. R., Equations of compatibility for finite deformation of a continuous medium,** *Amer. math. Monthly* **57,** 679–681, 1950.

3496. **Vigness, I., Transverse waves in beams,** *Proc. Soc. exp. Stress Anal.* **8,** 2, 69–82, 1951.

The encastre end of long flexible cantilever is suddenly accelerated. Free end motion was recorded by stream photography. Bonded gages measure strains at ten stations from root to tip. Beam takes up wavy form about initial undeformed line with waves of greatest amplitude and length near root source of shock. Whole beam gradually takes on general motion. Propagation velocity of inflection points is deduced. Maximum strain at root has a value 3.8 times initial constant strain. Simple Bernoulli-Euler theory appears to fit observations fairly well.

K. H. Swainger, England

3497. **Kucharski, W., Contribution to the theory of the plate loaded under uniform shear** (in German), *Ing.-Arch.* **19,** 1, 22–30, 1951.

In this third note, author studies the propagation of elastic waves in a rectangular thin plate submitted to shearing forces at the boundary. Equation of the inflection is then $w_{tt} + \Delta\Delta w - \vartheta w_{xy} = 0$ [for notations, see first note in *Ing.-Arch.* **18,** 1950]. Putting $w = f(x\cos\alpha + y\sin\alpha - ct)$, solution is obtained in the form $f^{IIII} + (c^2 - \vartheta\sin 2\alpha/2)f^{II} = 0$. Various types of waves are possible (progressive and standing). With the condition $\vartheta = \text{const}$, author obtains the simplest solutions; some of standing waves have a parallelogram form. Knowledge of the form of waves and propagation velocity allows one to deduce the structure of the plate. Some conditions of instability are also determined. Guilio Supino, Italy

Experimental Stress Analysis

3498. **Frocht, M. M., assisted by Sevin, E., Isopachic patterns and principal stresses in bars with deep notches in tension,** *Proc. Soc. exp. Stress Anal.* **8,** 2, 171–186, 1951.

Principal stresses are determined along the lateral axis of symmetry of bars in tension containing deep, sharp notches. Difference of the principal stresses is obtained from photoelastic observations and is not discussed here. Sum of the principal stresses is obtained by a numerical solution of Laplace's equation by combining a conformal transformation with an iteration procedure. Figures show principal stresses in four bars in which the radii at

the base of the notches vary from $^1/_{32}$ in. to $^1/_{16}$ in. and the depths of the notches vary from $^1/_4$ to $^1/_6$ the widths of the bars. Other figures show isopachic curves (lines of constant principal stress sum) for the same bars. A short bibliography is included.
William B. Stiles, USA

3499. Yuassa, K., and Yamamoto, K., An analysis of three-dimensional stress by crystal optics (in Japanese), *Trans. Soc. mech. Engrs. Japan* **15**, 50, I-34-50, Oct. 1949.

Paper gives an experimental method for analyzing three-dimensional stresses by means of crystal optics using the freezing method of a phenolite model. The first part describes the nature of two sorts of initial stresses which come into the model during the freezing experiment due to the instability of the material, and shows a correction method for finding the true value of stress, eliminating the initial stresses. The second part contains a concrete description of the experiment with freezing method and a derivation of formulas for three-dimensional stress analysis after the experiment. A convergent polarized monochromatic light is passed through a slice which is cut off from the test model; an interference fringe figure is photographed, and the values and directions of the principal stresses are analyzed from the figure by the aid of the formulas. The last part is a verification of the method by some examples of tests under known stresses. After freezing uniaxial and biaxial stresses in phenolite pieces, slices in various directions are cut off from the test pieces and experimental stress values are compared with theoretical ones. Results are quite satisfactory. Teruyoshi Udoguchi, Japan

3500. Soete, W., The strain gage (in Dutch), *Tech. Wet. Tijdschr.* **20**, 4, 71–75, Apr. 1951.

This general review of electrical wire-resistance gages precedes papers by VanCrombrugge and Colle on applications. Author discusses principles of gage, choice of wire material, types of gages, bonding of gages to test piece, and determination of principal strains and stresses. F. J. Plantema, Holland

3501. Box, W. A., The effect of plastic strains on stress concentrators, *Proc. Soc. exp. Stress Anal.* **8**, 2, 99–110, 1951.

Tests of 24S-T aluminum plate with circular-hole discontinuities of five types are reported. Strains at the holes were measured under slow loading to failure. Strain concentrations (ratio of maximum local strain to strain corresponding to P/A) increase in plastic region to point of failure. Stresses at holes when above elastic limit are taken from tensile stress-strain curve and measured strains at hole. Stress concentrations begin to decrease from theoretical values at elastic limit and approach unity before failure. Henry A. Lepper, Jr., USA

Rods, Beams, Shafts, Springs, Cables, etc.

(See also Revs. 3529, 3533, 3568)

3502. Amaral, C., A generalization of the Cross method (application to continuous circular beams) (in Portuguese), *Rev. Ord. Engr.* **8**, 75, 120–142, Mar. 1950.

Author refers to the only known application of the Hardy Cross method to structures in torsion, as developed by Correia de Araujo for structures consisting of straight members connected under right angles, and presents his method to continuous circular beams. The bending moments are distributed to the adjacent spans by using both flexural and torsional rigidity factors K_f and K_t which are derived from the principle of least work. The method is demonstrated on two examples with two and three spans, respectively. J. J. Polivka, USA

3503. Amaral, C., Fixed-end moments of circular beams (in Portuguese), *Rev. Ord. Engr.* **8**, 79, 299–311, July 1950.

In this addition to his previous paper, the author develops the formulas for fixed-end moment of circular beams under torsional loadings: (1) Uniformly distributed load. (2) Two symmetrical concentrated loads. (3) Concentrated load at the mid-point of the circular span. (4) Asymmetric concentrated load. (5) Load uniformly distributed over a section of the circular beam. (See preceding review.) J. J. Polivka, USA

3504. Holmberg, Å., Shear-weak beams on elastic foundation, *Publ. inter. Assn. Bridge Struct. Engng.* **10**, 69–85, Nov. 1950.

Beams with large shear deformation and resting on an elastic foundation are treated. Expressions for the moments, shears, and deflections are derived. A relation between the flexural rigidity, the shear rigidity, and the foundation modulus is established as a criterion for the correctness of this analysis. Applications to elastically supported Vierendeel trusses and continuous beams supported and rigidly connected to columns on elastic foundations are demonstrated. C. C. Wan, USA

3505. Niemann, G., Elements of machines (Maschinenelemente), vol. I, Berlin, Springer-Verlag, 1950, viii + 308 pp., 795 figs. DM 28.50.

Julius Springer books are usually well edited and contain numerous expertly drawn sketches and graphs. This book is no exception to these general standards; perhaps some lower quality of paper and of binding may be noticed. This volume is made up of four main parts: Fundamentals; elements for connection; bearings; and shafting. Fundamentals are presented in the first 60 pages and begin with a short note on the art of construction of machine elements followed by considerations of economy, service, and safety. General remarks are made with regard to the technology associated with welding, casting, pressing, as well as with various basic types of machining. Considerable space is devoted to the problems of wear and corrosion, which is a rather new and welcome addition to textbooks on machine design. Elementary strength of materials is reviewed and supplemented with a discussion of failures due to fatigue, vibration, and impact. One chapter is on lightweight constructions incorporating lightweight materials and designs, such as used in aviation industry. Concluding the first part is a chapter on engineering materials, describing the main physical and chemical properties of ferrous, nonferrous, and nonmetallic materials. The second part of the book describes various methods of connections such as obtained by welding, soldering, and by the use of rivets and bolts. A comprehensive discussion of all practical types of springs, with a short description of elements employing rolling and sliding contacts, concludes the second part of the book. The two last parts are devoted to bearings and shafting. Ball and roller bearings are discussed with special reference to loading and life limits. The performance of journal bearings is described as based on the hydrodynamic theory of lubrication together with a review of bearing materials and lubricants used. Shafting and shaft connections conclude the book.

The volume is well illustrated and is supplemented with numerous tables and graphs, all of which will be of value to students and to designers of actual machine elements. There are few, not too many, working examples. Each chapter has a rather impressive bibliography which is abundant with references of the last ten to fifteen-year period. However, among the many German references (over 650) there are only eleven American references.
L. M. Tichvinsky, USA

Courtesy of Journal of Applied Mechanics

3506. Wuest, W., Theory of the high-pressure measuring tube bored eccentrically (in German), *Ing.-Arch.* **19**, 1, 12–21, 1951.

O. Hoffmann and W. Francke, having discovered that eccentrically bored straight tubes are subject to bending moments under the influence of internal pressure, have patented [German patent 743 613 (1937), French patent 846 870 (1938)] a device for measuring high pressure (in excess of 1000 atm) based on this principle. Advantages claimed over the conventional Bourdon tube are lower peak stresses in the measuring device and ease of providing operator protection by incasement within an outer tube.

After pointing out inadequacies of existing theories [Bricas, M., "La théorie de l'élasticité bidimensionelle," Athens 1936; Föppl, L., *Drang und Zwang*, **3**, p. 145, Munich 1947], author presents a precise elastic analysis for eccentric thick tubes, giving explicit formulas for both deflection and controlling stress. Optimum conditions, i.e., maximum deflections under given limiting stress, are obtained for a ratio $e/(r_o - r_i) = 0.6$, where r_o and r_i are the outer and inner radius, respectively, and e is the eccentricity, or distance between the center lines of the two radii.

Results are given of six experimental deflection determinations for measuring tubes designed for pressures up to 120,000 psi; good agreement with theory is shown to exist on the assumption of a Poisson ratio of 0.25. A. R. C. Markl, USA

3507. Wada, S., The fluctuations of the power and torque caused during the gearing process of a pair of conjugate teeth in spur gears, *Memo. Fac. Sci. Engng. Waseda Univ. Tokyo*, no. 14, 32–33, 1950.

Paper derives expressions, taking into account tooth friction, for torque ratio and normal tooth loads on a gear tooth pair of unit contact ratio for the case of a path of contact of second degree and, hence, as a special case for involute gears. Some serious misprints make the paper difficult to follow, and author does not sufficiently emphasize that his results apply only at the beginning and end of contact and not intermediately. In reviewer's opinion, the form of analysis and the results offer no advantage over standard methods (cf. Buckingham's "Analytical mechanics of gears," p. 396). Ewen M'Ewen, England

Plates, Disks, Shells, Membranes

(See also Revs. 3497, 3518, 3527, 3528, 3533, 3540, 3582)

3508. Haas, A. M., An analysis of flat slabs, *Concr. Constr. Engng.* **66**, 4, 101–109, Apr. 1951.

An analysis is made of a continuous floor slab which includes a network of equally spaced column heads. The plate equation in polar coordinates is employed to analyze the column head when the latter has a variable stiffness proportional to $1/r^3$. At the circular junction of the column head and the floor slab the necessary continuity conditions are determined at a finite number of points. Some changes which are desired in the British Standard Code are discussed. D. L. Holl, USA

3509. Prusakov, A. P., Basic equations of bending and stability of three-layer plates with a light filler (in Russian), *Prikl. Mat. Mekh.* **15**, 1, 27–36, Jan.-Feb. 1951.

Bending and stability of a rectangular three-layer plate; outside layers of the same material, solid and heavy, the inside of a light nonisotropic filler. In contrast to other works of this kind, one considers a finite plate, where the filler suffers deformations and the stresses in both outside layers vary along their thickness.

Integrating the basic equations, author has 18 unknown functions of x, y. The conditions on top and bottom surfaces, as well as requirement of continuity of stresses and displacements in the areas of contact between filler and both outside layers, reduce the problem to a system of 6 partial equations only. Here its structure of a symmetrical and antisymmetrical part may wonderfully be seen.

Special attention is given to the formulation of boundary conditions for the most important kinds of support. At the end it is shown (also in an example) how to apply the general results to investigation of the stability of a three-layer plate.

Reviewer considers this original and correct paper enriches the theory and shows elegant mathematical work. Perhaps it would be useful to express final solution for the separate layers by the author's functions η_1, ξ_1, w_1, and η_2, ξ_2, w_2 from the equations (1.34) and (1.35). V. Vodicka, Czechoslovakia

3510. Giovannozzi, R., Study of thermal stresses in conical disks, in disks with constant thickness and in disks with arbitrary profile, by decomposition in partial trunks (in Italian), *Aerotecnica* **30**, 6, 308–315, Dec. 1950.

Author extends Honneger's method for computing centrifugal stresses in conical rotating disks by considering effects of temperature of type $T = a + br + cr^2 + dr^3 \ldots$ (a, b, c, d, constants, r radius of disk). Using superposition principle, which he bases mathematically on Stodola's general equation of stresses in rotating disks with thickness and temperature functions of radius, he separates centrifugal and thermal effects and obtains expressions for thermal stresses in terms of functions of a nondimensional radius. A table and graph of these functions' values for Poisson's ratio, $\nu = 0.3$, are given. A more accurate recalculation of Honneger's coefficients for centrifugal stresses is also included. For the special case of constant thickness and temperature variation $T = T_0 + k_n r^n$ (k_n, n, constants) the thermal stresses have simple expressions, not explicitly visible in a paper by Leopold [AMR **2**, Rev. 174], where formulas are given with centrifugal and thermal stresses combined. Application of these results to disks of arbitrary thickness variation is discussed by dividing the disk either in trunks of constant thickness or conical trunks, with a temperature either constant or linear for each trunk. An example treated by Leopold is reconsidered by means of these alternatives. Reviewer believes paper has practical value for designers of steam and gas turbines. G. A. Zizicas, USA

3511. Ancora, R. B., Analytical problems connected with the theory of the simply supported elastic plate (in Italian), *R. C. Semin. mat. Univ. Padova* **20**, part 1, 99–134, 1951.

Applying the methods developed by Picone and his co-workers, author proves the existence of the solution for the problem: $D\nabla^4 w = 0$ in the domain, $\nabla^2 w = 0$ on the boundary, when the boundary presents any kind of singularity, and evaluates an upper bound of error involved in the solution by Picone's method.

The center deflection of a simply supported square plate is evaluated by two separate methods and checked with Nadai's solution. M. G. Salvadori, USA

3512. Hopkins, H. G., Plastic deformation of rectangular plates under direct loads, *Engng. Structures*, New York, Academy Press, Inc.; London, Butterworth Science Publ., 1949, pp. 93–108.

In author's words: "A plastic flow theory is given of the small deformation of rectangular plates under uniform direct loads applied to all edges. . . The general problem is separable into three subsidiary problems. The first (von Kármán) problem is such that, immediately after normal displacements occur, the

applied loads are identical with those immediately before normal displacements occur. The second (Engesser) problem is such that, immediately before normal displacements occur, the applied loads assume least possible values if their (infinitesimal) increase is permitted when normal displacements occur. The third (Shanley) problem refers to the development of plastic deformation after normal displacements occur at the Engesser load. The von Kármán problem presents no serious mathematical difficulties, and the calculation of von Kármán loads in any particular case is largely a matter of computation. The Engesser problem appears, in general, to be more difficult; however, the calculation of lower bounds to the Engesser loads is similar to the calculation of von Kármán loads. The Shanley problem is very difficult, and the approximate solution of a simple case only may be possible. The von Kármán problem is solved, and the analysis is specialized to particular types of boundary conditions."

Author spends some time reviewing controversy between flow and deformation theories of plasticity. He postulates that recent experimental results which agree better with deformation than flow theory may be explainable by confusion between von Kármán and Engesser problems. Since only von Kármán problem is dealt with in detail, this remains conjecture. Recent slip theory of Batdorf and Budiansky, which has been applied to buckling plates, is not mentioned.

Philip G. Hodge, Jr., USA

3513. Wu, M. H. L., Linearized solution and general plastic behavior of thin plate with circular hole in strain-hardening range, *Nat. adv. Comm. Aero. tech. Note* 2301, 41 pp., Mar. 1951.

Solution is based on the deformation theory for finite strains. The instantaneous dimension of an element is considered in the analysis. Variation of octahedral shear stress-strain curve of the material is used as the criterion of the deformation theory in the strain-hardening range. The ratio of the principal stresses in this case has been shown by author to be essentially constant during loading. With plastic strain much more than the elastic strain, the condition of incompressibility is used. From the conditions of equilibrium and compatibility and with the transformation of radial and circumferential stresses in terms of octahedral shear stress and a parameter α, two nonlinear differential equations are obtained with the octahedral shear strain and the parameter α as dependent variables. These two equations are linearized by replacing the nonlinear terms with a simple linear term involving a constant which is determined by minimizing the effect due to the linearization.

Numerical results show good agreement between the linearized method and the solution without linearization. With linearization, amount of computation required is much reduced. Furthermore, tables and curves are presented to facilitate calculation of the distribution of octahedral shear stress and strain as well as the principal stresses and strains.

T. H. Lin, USA

3514. Mezhlumyan, R. A., Boundary conditions for bending and torsion of thin-walled shells beyond the yield point (in Russian), *Prikl. Mat. Mekh.* **14**, 5, 537–542, Sept.-Oct. 1950.

Continuing an earlier work [title source, **14**, 3, 1950], author uses formulas of the book of Il'yushin on plasticity (1948) to derive the conditions of equilibrium and the boundary conditions, as given by the minimum of the sum of the total energy of deformation and the energy of unprevented torsion, lessened by the work of the external forces. Both the ends free and not free from external forces are considered.

H. Schlechtweg, Germany

Buckling Problems

3515. Bültmann, W., Buckling strength of a straight column with variable compressive load and elastic clamping (in German), *Bautechnik* (*Stahlbau*) **28** (**20**), 4, 50–52, Apr. 1951.

Assuming a linear compressive load, the buckling problem of a straight rod for small deflections is reduced to a differential equation of the third order: This equation is transformed into a Bessel's equation and the solution is completed with the help of the Bessel's functions $J_{1/3}$, $J_{-1/3}$, $J_{2/3}$, $J_{-2/3}$. In two numerical examples the buckling load and the coefficient of effective length are obtained.

B. R. Seth, India

3516. Nowacki, W., The stability of a continuous slab subjected to longitudinal and lateral compressing forces (in Polish), Jubilee Book, M. T. Huber, Gdansk-Polit., pp. 305–316, 1950.

Author examines the stability of a continuous slab freely supported round its circumference, which is compressed simultaneously by longitudinal and lateral forces. From the assumption of the continuity of the slab's medial supports, the author derives a system of homogeneous equations; by assuming the determinator of this system to be zero, the sought criterion of buckling is obtained.

J. Mutermilch, Poland

3517. De Bruine, J., The influence of welding stresses on buckling (in Dutch), *Ingenieur* (**63**), 15, B.39-B.41, Apr. 1951.

Buckling tests of a welded and a riveted box-girder section 58 × 60 cm, length 6 m, thickness 11 mm. Plastic deformation appeared in the welded girder for a compression stress of 19 kg/mm², while 22 kg/mm² were attained in the riveted girder. Author explains the difference of residual stresses in the welded girder which were measured in a previous test and found to be 4 kg/mm² in the plate.

W. Soete, Belgium

3518. Bijlaard, P. P., Analysis of the elastic and plastic stability of sandwich plates by the method of split rigidities. I, *J. aero. Sci.* **18**, 5, 339–349, May 1951.

Author gives a simple theory for the problem of elastic and plastic stability of sandwich plates (thin foils with a connecting intermediate layer of a light material, used as light plates in aeronautical design). Deformations of the foils and their intermediate layer are assumed to follow a sinusoid. Further, author makes certain simplifying assumptions (deformation by shear being negligible in the foils, simplification of conditions in the contact zone between foil and intermediate layer, etc.) and finds a simple approximate equation between the buckling load of the whole plate and Euler's buckling loads of foils and layer. The same method is used to set up approximate formulas for plastic deformation. The parameters are derived from previous papers. He also gives a synopsis of formulas for application and diagrams relating to the elastic and plastic buckling of twelve different cases.

H. Neuber, Germany

Joints and Joining Methods

(See also Rev. 3562)

3519. Stern, E. G., Nails and screws in wood assembly and construction, *For. Prod. Res. Soc.* no. 180, 30 pp., May 1951.

A résumé of both technical and economic aspects of the use of plain-shank and threaded-shank nails vs. wood screws is presented in order to summarize our present knowledge and recent developments and to give an outlook of what is immediately ahead of us in the fields of wood-product assembly and wood construction with nails and screws.

As a result of the recent advance, especially in the design and mass production of threaded-shank nails, it should be possible to up-grade products assembled with these nails and to improve the efficiency in manufacturing and construction. A detailed bibliography on nails and nailing is included in the paper.

From author's summary

3520. Carlson, I. H., and Black, W. S., Joints for high-pressure high-temperature piping, *Trans. Amer. Soc. mech. Engrs.* **73,** 3, 237–246, Apr. 1951.

Paper outlines briefly the difficult problem of obtaining leak-proof joints for high-pressure high-temperature piping. It includes results obtained from an extensive test on three full-sized joints for connecting ferritic-alloy main steam piping to austenitic-alloy turbine equipment of 150,000-kw capacity.

From authors' summary

Structures

(See also Revs. 3454, 3517, 3560, 3566, 3589, 3655)

3521. Efsen, A., Influence lines for straight beams with variable moment of inertia (in Danish), *Frandsen Anniv. Vol. Lab. Bygn. Tekn. Medd.* no. 1, 37–47, 1950.

Paper gives simplified method for obtaining influence lines for moments at supports of continuous beams with haunches. Effect of haunches is approximated by using parabolic arcs in modifying moment diagram. Method is illustrated by an example. Tables aiding computations are given for 19 simple types of load. Application of method to arbitrary moments and shear forces is discussed briefly. Paper is best understood if read in connection with author's previous work ["Primaermoment-Metoden," Copenhagen, Jul. Gjellerups Forlag, 1950].

Tage A. Mortensen, USA

3522. Bignoli, A. J., The practical value of structures calculations (in Spanish), *Cienc. y Tecn.* **116,** 585, 121–138, Mar. 1951.

Paper deals with relationship between real structure and the representative sketch on which calculus is based. Author shows importance of adequately choosing this representative sketch and selecting the magnitudes which are to be considered as unknowns in statically indeterminate structures. Different criteria regarding factor of safety are given. The various methods of resolution of statically indeterminate systems are considered from a higher point of view, showing how all of them are basically only different ways of resolving systems of simultaneous linear equations. It is, therefore, convenient to resolve each problem by the method best adapted to it. Francisco G. Olano, Argentina

3523. Ferguson, P. M., Equilibrium equations without restraining forces for moment-distribution-sway problems, *Univ. Texas Bur. Engng. Res.* Circular 13, 37 pp., 1950.

Author proposes method for analysis of frames involving sidesway by moment distribution in which the step for determining value of restraining force is omitted. Method applies equilibrium equations directly to structure in order to measure initial unbalance. By the addition of corrective moments to the initial moments used in first application of the equilibrium equation, statics is satisfied. Initial moments and corrective moments are determined by moment distribution. Examples of rectangular frames, gabled bents, and unsymmetrical frames are included in bulletin. Author claims that method is teachable and easier to learn by students. Cameron M. Smith, USA

3524. D'Heygers, O., Method for calculating upper wind-bracings whose two booms are arcs without a vertical frame (in French), *Ann. Trav. publics Belg.* **104,** 2, 219–240, Apr. 1951.

Paper deals with an approximate method of calculating wind bracing in bowstring bridges, based on several assumptions, including substitution of shape of arch by three straight bars. Reviewer believes that this method is not so practical nor as approximate as the one previously indicated by Dischinger in Schleicher's Handbook (p. 1512), using Jakobsen-solved integrals for different shapes of arches. Arturo J. Bignoli, Argentina

3525. Massonnet, Ch., Design of bridge floors with multiple beams and their torsionalr esistance (in French), *Publ. inter. Assn. Bridge Struct. Engng.* **10,** 147–182, 1950.

Author refers to his previous paper on design of this type of bridge floors without considering torsional resistance, and generalizes Guyon's method [*Ann. Ponts Chauss.*, 1947] based on the assumption of a continuous grid of longitudinal and transverse beams, for the special case of the torsional resistance of structural members. Numerical values of the transverse distribution coefficients of the loads for all bracing parameters and for various values of torsional stiffness of the beams are tabulated, which makes it possible to calculate the characteristic coefficients of the bending moments in the transverse beams. Practically, the method may be adopted like that of Guyon, if it is restricted to the first term of Fourier's series; it thereby becomes possible to take the torsional resistance of the bridge into consideration without additional calculations. If necessary, it can also determine exactly the influence of any load by developing into a Fourier series. The method furnishes not only more accurate results than the classic methods in case of typical one-span bridges, but can also be modified and used for bridges with continuous beams prestressed transversely, and for bridges in reinforced and prestressed concrete of greater economy. In the theoretical part, reference is made to Timoshenko's "Theory of plates and shells," to research of Illinois Engineering Experimental station, and to papers by Richart, Newmark, Spiess, and Leonhardt.

J. J. Polivka, USA

3526. Scivey, J., The estimation of wing weight, *Aircr. Engng.* **23,** 267, 143–144, May 1951.

Paper suggests a means of estimating wing weight using formulas based on the theoretical ideal wing. The predicted wing weight is attained by multiplying the weight of the ideal wing by a correction factor derived from a statistical analysis of actual wing weights. Method provides the designer with a yardstick by which to compare alternative designs and indicates where improvement might be effected.

From author's summary by T. H. H. Pian, USA

3527. Garvey, S. J., The quadrilateral "shear" panel, *Aircr. Engng.* **23,** 267, 134–135, 144, May 1951.

One important problem of sweptback wings is the stiffness of nonrectangular shear panels. Assuming stable conditions, author introduces a state of stress in a quadrilateral panel that satisfies the equations of equilibrium but not those of compatibility. The boundary conditions are satisfied "in the mean," i.e., resultant shear forces at the free edges are prescribed. From the state of stress author deduces the *stress energy* stored in the panel, which gives a measure of the stiffness.

Reviewer points out that since the equations of compatibility are not satisfied, the so-obtained stress energy (and stiffness) is

greater than the actual. But the error is probably very small for nearly rectangular panels, and the uncertainty of the boundary stresses in actual constructions should introduce much greater errors.

The problem of replacing such a panel by "diamond" bracing is also treated. Frithiof I. Niordson, Sweden

3528. Leggett, D. M. A., and Chapman, R. G., Distortion of control surface panels, *Aero. Res. Counc. Lond. Rep. Mem.* 2478, 30 pp., Nov. 1944, published 1951.

Authors describe theoretical and experimental work on deflection of control surface panels with elastic side supports under typical air-loads distribution. Experimentally measured distortions of typical ailerons and elevators of World War II aircraft are predicted by simple theory with reasonable accuracy.

George Gerard, USA

3529. Steinhardt, O., On calculation of compound trusses (in German), *Bautechnik, Stahlbau* **28 (20), 1, 9–12, Jan. 1951.**

Article deals with secondary moments and stresses in bridge or roof trusses in which the supported concrete slab is considered as part of truss chord. The effects of the eccentricity of the slab with respect to the steel chord center line and of thermal expansion are taken into account. Several methods of analysis are indicated. Brief discussion of Visintini vs. Vierendeel trusses is included. A. D. Topping, USA

3530. Wang, T. K., General instability of semimonocoque structures under combined loads of bending and compression, *Sci. Rec., Peking* **3,** 1, 121–133, Oct. 1950.

Paper is an extension of author's previous analyses for compression [AMR 1, Rev. 67] and bending [*J. aero. Sci.* 13, 1, 29–37, Jan. 1946]. General instability means simultaneous buckling of rings, stringers, and skin. Analysis is applicable to elastic circular cylinders having identical equally spaced rings and stringers. Rings are assumed to buckle only in their plane and without extension. Using Fourier series representation for buckling shape deformations, critical stringer load is calculated by equating external work to strain energy of buckling due to ring and stringer bending and torsion plus skin shear. Charts of critical stringer load in terms of certain elastic and dimensional parameters are included for ratios of bending stringer force to compression stringer force of $^1/_2$, 1, and 2.

C. M. Tyler, Jr., USA

3531. Tschech, E., and Jaburek, F., Static model experiments for arched dams (in German), *Schweiz. Bauztg.* **69; 16,** 17; 215–219, 235–239; Apr. 1951.

Paper presents diagramatically the results of plaster-celite model tests of the Salza (symmetrical) and Hierzmann (nonsymmetrical) arch dams in Austria and compares these results with those obtained by trial-load analysis. Reasonably good agreement between the two methods is found at some points but not all. Author infers that this is due to taking an insufficient number of sections in the trial-load analysis to include the effect of unusual curvature of the cantilever elements near the base. Model material and measuring methods are similar to those used by the U. S. Bureau of Reclamation. Comprehensive treatment of these points and also the trial-load method of analyzing arch dams can be found in the Boulder Canyon Final Report Series issued by the U. S. Bureau of Reclamation. Most striking departure from usual practice in this type of test is the use of 50 concentrated loads instead of a continuous load. Author points out that this gives a discontinuous loading, but believes that the possibility of loading in increments outweighs the disadvantage. Reviewer notes that this type of experimentation is valid for only a limited number of loading conditions, and that only calculation gives a complete analysis. Principal value of paper lies in the numerous diagrams which can be used for comparison.

J. R. Bruggeman, USA

3532. Kanai, K., A method of determining the stiffness of each story of a *n*-storied building, *Bull. Earthq. Res. Inst. Tokyo Univ.* **28,** parts 1–2, 161–166, Jan.-June 1950.

Short paper (including note on Kanai's formula by Ryutaro Takahasi) gives resonance equation solved for the stiffness of each story in terms of observed mass, displacements, and period of vibration. Equation is applied to two actual buildings, and shows immediately the different characteristic behavior in regard to end fixity of the columns between fundamental and second mode. Jacques Heyman, England

3533. Baker, A. L. L., Recent research in reinforced concrete, and its application to design, *J. Instn. civ. Engrs.*, no. 4, 262–298, Feb. 1951.

A general plastic theory for the ultimate strength of statically indeterminate reinforced-concrete framework is presented, based upon the insertion of n hinges for a case n times statically indeterminate. These hinges must be located to correspond to points where plastic hinges will develop under the given type of loading. When the moment necessary to produce plastic hinge action at each hinge is known, the system is statically determinate and easy to calculate. The angle of rotation at each hinge can be calculated without the solution of simultaneous equations.

Such frame calculations for reinforced concrete require knowledge of the ultimate strength of individual members, the limiting EI values for elastic members, and the limiting angles that can be developed at a plastic hinge. Formulas for ultimate strength and limiting angle are developed both for ordinary and prestressed concrete. Experimental work is under way to evaluate the several limiting constants involved. To this end, a special bending-simulation machine has been constructed, capable of producing any desired stress distribution on the beam cross section. Short sections can be tested instead of full length beams. Preliminary values of constants thus evaluated are given. Later results, promised in an addendum to the paper, should remove important uncertainties now present in ultimate strength calculations.

A plastic theory for the ultimate longitudinal bending strength of cylindrical shells is also presented and laboratory tests on large scale models are mentioned. Results from a pre-stressed shell now under test will be reported later. Phil M. Ferguson, USA

3534. Wilkins, R. J., Some experiments on the load distribution in bond tests, *Mag. Concr. Res.* no. 5, 65–72, Jan. 1951.

Tests are described briefly in which the bond-stress distribution has been obtained for tubes embedded in concrete and used as "pull-out" specimens. Electrical-resistance strain gages were inserted in straight lengths of steel tubing to form the measuring devices which can be used for direct calibration against load. These tubes were then embedded along the axis of cylinders of concrete to form pull-out specimens of the usual type. Readings of the strain gages, taken as the tube is being withdrawn from the concrete, enable the load distribution curves to be plotted. The variation of bond stress has been studied from the information provided by the load distribution curves, and values are given for average and maximum bond stress for various embedded lengths and types of tube surface. Results obtained from an American bar of "Hi-bond" type with a patterned surface are included.

The experimental method has proved to be satisfactory in use

and capable of providing sufficient data for plotting load-distribution curves of reasonable accuracy. Transition of the load from the end at which it is applied to the free end of the tube has been observed, and the method enables the mechanism of bond failure to be investigated. The most important influence appears to be the character of the contact surface, and it is considered that the tests indicate that bond may be caused by (a) adhesion, (b) friction, and (c) mechanical wedging. Relative importance of these factors is apparently controlled by the type of surface, but further work on this subject is in progress.

From author's summary

3535. Price, W. I. J., and Trott, J. J., Multi-channel apparatus for recording temperatures, *J. sci. Instrum.* **28,** 1, 15–17, Jan. 1951.

Apparatus described was developed for an investigation of the temperature gradients in various road surfacings and subgrades, an experiment involving continuous recording from sixty thermocouples. These are arranged in two groups of thirty, and each thermocouple connected in succession at $^1/_2$-min intervals with a chopper unit rendering the emf intermittent. A selector mechanism is used to connect the thermocouples, and the intermittent emf's resulting from chopping are fed to a conventional resistance-capacity amplifier, rectified, and then fed to the appropriate channel of a two-channel pen recorder. This arrangement records the readings of each thermocouple every half hour for continuous periods up to 100 hr. From authors' summary

3536. Neumann, G., Who invented prestressed concrete (prestressing)? (in Italian), *Ind. Ital. Cem.*, Mar. 1951.

Dr. Neumann, one of the leading engineers in Italy connected with the idea of prestressing since early practical applications, reviews developments since 1886 when the San Franciscan, P. H. Jackson, originated this idea, followed by Mandl (1896), Rabut (1904), Koenen (1907), Vianini (1914), Wettstein (1919), Emperger (1922), Ruml (1929), and then by a number of well-known specialists (Freyssinet, Dischinger, Magnel, Finsterwalder, etc.). Special reference is made to the paper "Patents and codes relating to prestressed concrete" by Curzon Dobell in *Amer. Concr. Inst. J.*, May, 1950, and to Disc., Dec. 1950.

J. J. Polivka, USA

3537. Michalos, J. P., Effects of lateral loads on arches, *J. Amer. Concr. Inst.* **22,** 5, 377–387, Jan. 1951.

Influence lines for moment and shear in unbraced, parabolic arch ribs of constant cross sections are given for arches having several values of rise ratio and bending to torsional stiffness. The effect of haunching is studied. Procedures are presented for drawing approximate and exact moment curves in both braced and unbraced ribs. Numerical studies are included.

From author's summary by A. H. Finlay, Canada

3538. Swida, W., Calculation of steel arches with consideration of the bearing capacity in the elastoplastic state (in German), *Bautechnik (Stahlbau)* **27 (19), 28(20);** 11(3), 12(3), 2(2); 17–20, 29–31, 25–28; Nov., Dec. 1950, Feb. 1951.

Author develops expressions for the analysis of steel arch ribs stressed in the elastoplastic range. It is claimed that this method results in a simplification of the calculations in many cases, and, in addition, results in a more economical use of material. Paper is divided into three main parts: (I) Introduction, in which some elementary relationships of an idealized material are considered. (II) Elastoplastic state in a curved member subjected to bending and thrust, in which mathematical relationships are obtained on the basis of the common assumptions regarding plane cross sections and magnitude of deformations. (III) Calculation of steel arches, considering plastic deformations, in which expressions are obtained for moments and forces. Particular equations are included for several special cases involving parabolic and circular arches. James P. Michalos, USA

3539. Herrmann, C., Basic principles: Foundations of power hammers (in German), *Schweiz. Bauztg.* **69,** 11, 143–146, Mar. 1951.

Author analyzes problem, considering hammer, anvil, and foundation mass as rigid bodies coupled to each other. Impact of hammer on anvil is treated on basis of conservation of momentum and a coefficient of restitution. The anvil is assumed separated from the foundation by an elastic pad with damping. Author calculates maximum pressure on pad between anvil and foundation, maximum pressure of foundation on earth, and efficiency of hammer (proportion of useful energy to total energy). M. P. White, USA

3540. Hahn, L., Calculation of foundation plates (in French), *Ann. Ponts Chauss.* **120,** 6, 669–718, Nov.-Dec. 1950.

Neglecting the influence of the shear stresses, a method is presented for calculation of the stresses in foundation plates for ratios between the length and the height up to two. It is assumed that the force distribution on the upper and lower face may be expressed as a linear function of the coordinates x and y, and that these forces are transmitted from one face to the other by means of imaginary bars, filling up the total volume of the foundation plate. For the general case, a function is derived, by the aid of which the corresponding points for the passage of such a bar between the two faces can be determined. With this function, a number of special shapes in common use are studied, in particular, foundations with circular and rectangular faces under various loading conditions. R. G. Boiten, Holland

3541. Fornerod, M., Load and destruction test of 160-ft girder designed for first prestressed concrete bridge in U.S.A., *Publ. int. Assn. Bridge Struct. Engng.* **10,** 11–35, Nov. 1950.

Describes the first prestressed concrete bridge in U.S.A., designed on Magnel-Blaton system and constructed by adopting "post-stressing" methods. If designed on conventional lines as an open-spandrel arch bridge, it would cost some $300,000 more. Bridge has a central single span of 160 ft, and two approach spans of 74 ft. Main girders, 13 in number, are of I-section 6 ft 7 in. deep. Bridge was tested to destruction to obtain valuable information. Failure took place at total midspan moment of 20,800,000 ft lb, which was 2.75 times the total working-load moment. Since dead-load portion of working load is permanent and not subject to variation, it is excess of failure load over and above dead load which gives measure of safety against occasional overload. Test demonstrated that bridge could be loaded eleven times the standard live load before failure occurs.

Discussion is given of stress limitations and various design considerations including shrinkage, plastic flow and creep, and calculated stress diagrams drawn. Results of strain gage and deflection measurements are tabulated and compared with calculated values.

Tests revealed the importance of differentiating safety against cracking and safety against failure. The first can be calculated fairly closely on bases of stresses in homogeneous material, while second criterion is influenced by a radical redistribution of stresses in semi-elastic state of material, in which the shape factor assumes importance due to contributory influences of plastic deformation near failure load. To cover design of prestressed concrete by special code, author suggests considerable research in the field of semi-elastic materials and structures. S. K. Ghaswala, India

3542. Stüssi, F., The calculation of a Vierendeel girder (in German), *Publ. inter. Assn. Bridge Struct. Engng.* **10,** 199–210, Nov. 1950.

Because of its high degree of statical indeterminacy, the calculation of Vierendeel girders leads to a number of simultaneous equations which require a large amount of arithmetical work for their solution. In the proposed method, author considers the usual hinged frame girder as the basic system, and by an ingenious combination of the joint moments he points out that only a few of the redundant elements figure in each elastic equation so that the system can easily be solved.

Reviewer believes that this method could be applied successfully to other hyperstatic structures of high degree of indeterminacy. Aurel A. Beleş, Rumania

3543. Lash, S. D., A note on stress distribution in crane girders, *Engng. J. Montreal* **34,** 4, 310–312, Apr. 1951.

Author reports strain measurements on an aluminum alloy beam consisting of an I-section with a channel member fastened to the top flange. Strains agree well with values computed from formulas developed by the author, taking into account the fact that plane sections do not remain plane. Results indicate that maximum stresses can be computed with reasonable accuracy by theory of simple bending. John W. Clark, USA

3544. Kinzey, B. Y., Jr., Tests on the design of wood box columns, *Civ. Engng. Lond.* **46,** 535, 536, 537; 41–44, 108–110, 185–186; Jan., Feb., Mar. 1951.

Paper is a condensation of a thesis on "An investigation of wood box columns to determine formulas for their design (Virginia Polytechnic Institute Department of Wood Construction)" and is based on the results of tests on 102 columns. Constructional variables such as column slenderness, plank thickness, fastening, arrangement, and the use of spacers for individual plank stiffening were studied as to their effects on column strength. Design formulas for box columns are presented.

The saving in material and the wider variety of cross-sectional areas for box columns than is possible with standard timber sizes for solid columns are some of the economic aspects presented.
E. George Stern, USA

3545. Blaszkowiak S., Rhombus and combined latticed structures calculated by Hardy Cross method (in Polish), Jubilee Book, M. T. Huber, Gdansk-Polit, pp. 21–28, 1950.

The work contains an example of rhombus-latticed structure, with fixed joints, solved by the moment-distribution method. Author applies his own improvement consisting of one-stage solution, which enables the rotation and the sway of the joints to be taken into account simultaneously. J. Mutermilch, Poland

Rheology (Plastic, Viscoplastic Flow)

(See also Revs. 3512, 3513, 3538, 3561, 3563, 3571, 3572, 3728)

3546. Geiringer, H., On the plane problem of a perfect plastic body, *Grad. Div. appl. Math. Brown Univ. tech. Rep.* A11-55, 33 pp., 1950.

Author considers two plane plastic stress problems in this paper. First, for an arbitrary yield condition, the analogy between a proper form of the equilibrium relations of plasticity (or elasticity) and the relations between the stream and potential functions of a two-dimensional compressible flow lead the author to a study of the stress plane. This plane is determined by the variable S the mean stress, and θ the angle between the x-axis and a principal direction of stress in the physical plane. Essentially, this plane is the analog of the hodograph plane in the case of compressible fluids. The author notes that this analogy leads to the result that the yield condition corresponds to the adiabatic relation. Various formulations of the equilibrium relations in the stress plane are studied. All of these lead to linear second-order partial differential equations. It is shown that the character of these equations (elliptic, parabolic, hyperbolic) is determined by the yield condition. Secondly, author examines three particular yield conditions: (1) The von Mises "quadratic condition"; (2) the Saint-Venant "hexagonal condition"; (3) a new "parabolic condition" due to von Mises. In the hyperbolic domain, the characteristics for the general yield condition and in particular for the above three yield conditions are studied. Finally, author considers the case when one family of characteristics is a simple wave (an equivalent term is a "fan" or a "lost solution"). These simple waves and the cross characteristics for each of the above three yield conditions are explicitly determined. The above results are to be applied to problems in a future report. As the author notes, some of her results have been previously obtained [P. G. Hodge, Jr., same report series A11-52, 1950; von Mises, AMR **2,** Rev. 1135; Sokolovsky, *J. appl. Mech.* **13,** A-1–A-10, 1946].
Courtesy of Mathematical Reviews N. Coburn, USA

3547. Stüssi, F., Answer to the note of Prof. P. P. Bijlaard, (in German), *Z. angew. Math. Phys.* **2,** 2, 118–119, Mar. 1951.
See AMR **4,** Rev. 2916.

3548. Geiringer, H., Simple waves in the complete general problem of plasticity theory, *Proc. nat. Acad. Sci. Wash.* **37,** 4, 214–220, Apr. 1951.

Author considers problem defined by three stress components σ_x, σ_y, τ_{xy} and two velocity components v_x, v_y. These must satisfy two equilibrium equations, two stress-strain relations, and a yield condition. Stress-strain relations assume one form for plane strain, another for plane stress; yield condition is arbitrary, assuming only isotropy. For these general relations, author finds equations of characteristic curves and relations along these curves. Techniques for solving classical "initial value problems" are indicated. Exact solution is found for "simple wave region," containing one family of straight characteristics.

Paper is definitely mathematical, and no solutions of actual boundary-value problems are indicated. To reviewer's knowledge, reduction of actual plane-stress boundary-value problems to "initial value problems" is not known. Also, in only plastic plane-stress problem solved to date [Hill, AMR **4,** Rev. 209], flow does not remain plane, and thickness must be introduced as additional unknown. Philip Hodge, Jr., USA

3549. Edelman, F., On the coincidence of plasticity solutions obtained with incremental and deformations theories, *Grad. Div. appl. Math. Brown Univ. Tech. Rep* A11-49, i + 33 pp., 1950.

Author investigates certain conditions under which a uniform magnification of surface loads on a plastic body leads a corresponding proportional magnification of the interior stresses, and under which deformation theories may be substituted for incremental theories. It is shown that, for a large class of loading criteria, the desired situation obtains if and only if the stress-strain relation is of the so-called power-term type.
Courtesy of Mathematical Reviews F. B. Hildebrand, USA

3550. Pieruschka, E., The mathematical foundations of a method for measurement of the shear modulus of viscous fluid (in German), *Z. angew. Math. Mech.* **31,** 3, 83–92, Mar. 1951.

Equations governing flow of non-Newtonian fluid between con-

centric, oscillating cylinders are integrated to give formulas for shear modulus in terms of measured values taken from instrument of this type. The experimental results give examples of non-Newtonian liquids. John A. Lewis, USA

3551. Salt, D. L., Ryan, N. W., and Christiansen, E. B., The rheology of carboxymethylcellulose dispersions in water, *J. Colloid Sci.* **6, 2, 146–154, Apr. 1951.**

Relationships between stress and shearing rate are determined with a rotating cylinder consistometer on pseudoplastic dispersions of 0.67 and 1.01% of CMC in tap water in a range of 300 to 315 K, and found to be accurately represented by equations based on the relaxation theory of viscous flow. The latter are considered a suitable basis for developing reliable methods of predicting pressure losses in pseudoplastic flow of fluids in pipes.

Hans F. Winterkorn, USA

3552. Kê, T.-S., A new internal friction peak and experimental evidence of the existence of dislocations in metals, *Sci. Rec. Peking* **3, 1, 61–65, Oct. 1950.**

Internal damping measurements have been employed often in recent years to study relaxation-time effects. In particular, if a graph of internal dissipation as a function of frequency (or temperature) shows a peak, then this peak is supposedly connected with a relaxation time of the material under study. Present paper reports such a study in which it is believed the observed peak in dissipation is associated with the phenomenon of capture of dislocations by a solute atom. The material studied was pure aluminum with 0.5% of electrolytic copper added. It was annealed at 500 C and cold-worked by drawing through a 0.033-in. die, after which it was annealed for one hour at 300 C. The grains in this material were still imperfect.

It was found that the internal damping at any given frequency and temperature was a function of the amplitude of vibration and was a maximum at a specific amplitude. Likewise, the damping as a function of temperature at a fixed amplitude of vibration also showed a maximum. No peaks were found with high-purity aluminum which was copper-free or with the alloy after complete recrystallization had been effected by annealing at 400 C for two hours. Author interprets these results as substantiating the notion that dislocations and solute atoms tend to bind together.

Horace M. Trent, USA

3553. Drucker, D. C., Stress-strain relations in the plastic range—a survey of theory and experiment, *Grad. Div. appl. Math. Brown Univ.* **A11 S1, Dec. 1950.**

This section contains a summary of fundamental facts found by experiment. Detailed description is omitted. Purpose is to collect the salient points which any mathematical theory worthy of the name must possess. Obviously, such a procedure contains elements of danger. Probably the facts given are correct, but some may be omitted inadvertently. As in all fields, conclusions are in a sense tentative until many important experiments are performed and checked. From author's summary

3554. Crussard, C., A method of exact analysis of the shape of stress-strain curves. Application to the role of intergranular boundaries and impurities (in French), "Some rec. Develop. Rheol.," United Trade Press, London, 53–62, June 1950.

Stress-strain curves are represented in terms of the expression $\sigma = \sigma_0 + A\epsilon^m$. A method of differential analysis is given which allows determination of the factor σ_0 and m, even when both vary with stress. Application to experimental results obtained with polycrystalline metals shows the existence of three distinct regions of deformation, each of which is characterized by a set of constant values σ_0 and m. The main factor is, in the first region, the resistance of the grain boundaries; in the second one, both grain boundary and intracrystalline resistance; and in the third one, crystal fragmentation. B. Gross, Brazil

3555. McAdam, D. J., Jr., The stress-strain energy relationship for aluminum, *Trans. Amer. Soc. Metals* **43, 1215–1245, 1951.**

The effect of temperature on the flow stress involves a direct effect and an effect on the rate of increase of intrinsic strength with strain. The lower the temperature, the more rapid is the increase of intrinsic strength with strain. This effect of temperature on the true rate of work-hardening is much greater for aluminum than for copper, Monel, or nickel. With plastic strain, the direct temperature factors for aluminum and copper are practically constant and not very different for aluminum, duraluminum, copper, Monel, and nickel.

The influence of temperature on total strain energy involves two factors: (1) The energy corresponding to the direct temperature factor; (2) the work-hardening energy, the energy corresponding to the intrinsic strength factor. The total strain energy is the product of these two factors. With plastic strain, the latent energy increases continuously, at a decreasing rate; the ratios of the latent energy to the total energy and work-hardening decrease continuously, at a decreasing rate. For constant total strain, the ratio of latent energy to total energy varies little with temperature, but the ratio of latent energy to work-hardening increases as the temperature is lowered. The increase of latent energy with plastic strain is a function of the strain and temperature, not of the total strain energy.

Since the various diagrams representing the stress-strain energy relationship are based on the results of ordinary tension tests, these diagrams reveal no evidence of crystal recovery of cold-worked aluminum throughout a range from −188 C to room temperature. When aluminum has been strain-hardened at −188 C, the temperature range for softening due to crystal recovery extends far below room temperature. The excess intrinsic strength due to strain-hardening of high-purity aluminum at −188 C disappears almost entirely during a few days at room temperature.

From author's summary by E. Siebel, Germany

Failure, Mechanics of Solid State

(See Revs. 3574, 3582)

Material Test Techniques

(See also Revs. 3496, 3564)

3556. Fukuroi, T., and Shibuya, Y., The measurement of the Young's modulus of metals and alloys by an interferometric method. I. The Young's modulus of Ni-Cu alloys, *Sci. Rep. Res. Inst., Tohôku Univ. Ser. A,* **2, 5, 748–757, Oct. 1950.**

Authors describe their test equipment for determining the modulus of elasticity of bars by measuring the deflection due to very low static bending stresses. They present an experimental curve of Young's modulus of annealed Cu-Ni alloys vs. per cent composition. From 0 to 36% Cu, the curve is similar to a peroidic function having two maximum and one minimum points. From 36 to 100% Cu, the modulus decreases almost linearly. Authors compare their results with those of other investigators.

Alexander Yorgiadis, USA

3557. Felix, W., Nondestructive testing of large forgings with ultrasonics (in German), *Schweiz. Arch.* **17, 4, 107–113, Apr. 1951.**

Paper deals with practical experience gained by nondestructive testing of large forgings with the supersonic-flaw-detector, H. Hughes, Mark II b. Results of controlling compressor-rotors are discussed especially. Author refers to possibility of testing welded seams (with special search crystals), of controlling the adhesion between bearing bushings and bearing metal, or the amount of looseness in large crankshafts constructed by shrinking. Author states that ultrasonic testing already presents a very useful method for engineering practice. For testing large cross sections, where older known methods are failing, it will be particularly valuable. Caution is needed in interpreting results of testing, as relatively small defects are indicated with great sensitivity.

Hermann Möller, Germany

3558. Kochanovská, A., X-ray method for determination of anisotropy of crystals in polycrystalline substances, *Engng. Rev. Prague*, no. 3, 1–8, June 1950.

Author presents a method of determining anisotropy under internal or external stress of normally isotropic polycrystalline materials. Method depends on using x rays of different wave lengths so that reflection angles close to 90° can always be used, thus obtaining high accuracy. Discussion includes reference to cells of cubic structure. Method is useful because it is not necessary to grow large monocrystals. Experimental technique, sources of radiation, and data are given for α-brass, iron, and aluminum. Donald G. Ivey, Canada

3559. Kochanovská, A., X-ray method for measuring anisotropy of thermal expansion of metals, *Engng. Rev. Prague*, no. 4, 6–11, July 1950.

Author applies method he developed previously (see preceding review). He feels anisotropy of thermal expansion-coefficient measurements are useful not only for studying thermal behavior but also for measuring deviations from Hooke's law in different directions in a stressed material. Precision is obtained in the back reflection method by using different wave lengths of x rays in the different crystallographic directions so that reflection angles close to 90° are always used. Results are given for pure aluminum, technical aluminum, iron, and chrome steel. Measurable anisotropy is observed in all cases but the first. Mean expansion coefficients determined by this method are systematically less than those obtained by optical methods. This may be because the latter values depend on the binding material between individual microcrystals. There seems to be a close connection between thermal anisotropy and additional alloying of other elements for metals of cubic structure. Donald G. Ivey, Canada

3560. Thompson, J. N., Techniques used in the experimental stress analysis of reinforced concrete structures, *Proc. Soc. exp. Stress Anal.* **8**, 2, 111–116, 1951.

Author describes applications of SR-4 resistant wire strain gages. Of particular note is the ingenious use of these gages, mounted internally, to detect the strains of a reinforcing bar. Author accomplishes this application by planing two circular bars down to mating semicircular sections, milling a rectangular groove in each, mounting a number of gages in each, and tack welding the sections together to re-form the desired rod. Technique for fastening and protecting gages and leads is described in adequate detail. Author states that gages with $^1/_8$-in. or $^1/_4$-in. gage lengths were unstable under repeated or dynamic loadings and over extended periods of time. Gages with $^{13}/_{16}$-in. gage lengths were satisfactory under these conditions.

There is included a brief description of techniques found to be successful in the application of SR-4 gages to the surface of concrete and tile. M. J. Holley, USA

Mechanical Properties of Specific Materials

(See also Revs. 3490, 3544, 3555, 3586)

3561. Scotchbrook, A. F., Johnston, B. G., and Stout, R. D., Scatter in transition test curves, *Weld. Res. Suppl.* **16**, 5, 266s–271s, May 1951.

Descriptive information is given on the scatter of measurements of various brittleness criteria, using Lehigh test (0.01-in. root radius) and making ten tests at every 20 F, covering the whole transition range. From these data, "reliabilities" of transition temperatures are calculated, using a probability level of 98% and for 24 tests made (4 tests at each for 6 different temperatures, 60 F apart). Choosing a low ductility level as criterion (which has considerable merit), the transition temperatures for 1% contraction, 400 ft-lb energy and 50% cleavage have reliabilities of ±9.5, ±7.5 and ±19 F, respectively. On the half-maximum level of contraction and energy, these reliabilities are ±35 and ±39 F, respectively.

Reviewer thinks better reliabilities would have been found by using a more correct stochastical interpretation of the data; for instance, by drawing smooth curves for criteria and deviations and by taking into account that the scatter of a curve is smaller than the scatter of the individual means.

J. H. van der Veen, Holland

3562. Soete, W., and VanCrombrugge, R., Study of fatigue resistance of welded assemblies (in French), *Rev. Soud.* 6, 2, 72–82, 1950.

Authors study the problem of explaining the influence of residual stresses in welded joints of normal structural steel. A special fatigue-test specimen for investigating this influence was designed and the results from the special tests are compared with those from conventional tensile fatigue tests. It is difficult to obtain a distinct interpretation from this comparison, but the authors have decided to continue with the mentioned method in order to get more satisfactory results. Ragnar Nilson, Sweden

3563. Mitchell, K. W., Cumulative damage in fatigue, *Metallurgia, Manchr.* **43**, 255, 25–28, Jan. 1951.

Paper summarizes the present knowledge relevant to cumulative damage in fatigue. As there are many parameters in this subject, and not all investigators combine them in the same manner, the picture presented is, on the first reading, confusing. Paper throws no new light on the problem nor are any conclusions possible which would help the designer. It is, however, an admirable survey. It appears that the author has carried out no research in this field himself and one or two loose statements occur. For example, Owan's paper on fatigue (1930) should be consulted before stating: "It so happens, for reasons not yet explained, that there is a fairly definite relationship between the maximum stress which a material will withstand under repeated loading (stress concentration absent) and the 'tensile strength' as normally determined." J. A. Pope, England

3564. Gough, H. J., Engineering steels under combined cyclic and static stresses, *J. appl. Mech.* **17**, 2, 113–125, June 1950.

This paper is only one part of a larger study to be published in book form by His Majesty's Stationery Office (England). Author begins with the description of a new fatigue machine which can apply static bending stresses or static torsional stresses, or both, combined with reverse bending or reversed torsional stresses. A large number of tests were conducted with S65A steel and the results compared to several suggested criteria. Solid and hollow specimens, with and without holes, fillets, and splines were used.

Reviewer did not find in the paper the explanation of some of the symbols used. Although the author mentions the scatter of his results, no quantitative estimate of this scatter is given.

A. J. Durelli, USA

3565. Long, R. A., Dike, K. C., and Bear, H. R., Some properties of high-purity sintered wrought molybdenum metal at temperatures up to 2400° F, *Nat. adv. Comm. Aero. tech. Note* 2319, 75 pp., Mar. 1951.

Tensile properties of high purity, sintered, wrought molybdenum at temperatures from 1800 to 2400 F in protective atmospheres, and effects of swaging, recrystallization, and specimen size are presented. Data on stress-rupture properties of rolled molybdenum plate are also given. Metallography and fracture studies are included.

Outstanding tensile, ductility, and stress-rupture properties are reported, which are comparable or superior to temperature-resistant materials now in use. At 1800 F, tensile strength varied between 25,000 and 33,000 psi and 100 stress-rupture strength was 19,300 psi. At 2400 F, tensile strength varied between 13,000 and 26,000 psi. Tensile strengths were decreased by recrystallization. Degree of swaging and area of test section had little effect on tensile strength.

B. J. Lazan, USA

3566. Ahrens, W., Rigid splices for timber beams (in German), *Bautechnik* **27,** 1, 16–18, Jan. 1950.

Analogous to the theories used for the design of steel-plate girders, flexurally stressed nailed splice joints with side plates were analyzed whereby the following basic assumptions were made: The splice plate is relatively long in comparison to its height; the splice does not deform; and the deformation of the joint increases directly with load increase. On the basis of these assumptions, tabulations and nomographs were prepared according to which splice joints with a single continuous group or several groups of nails can be designed.

It was shown that 60% of the number of nails were necessary if nails were spaced in two separate groups in 8% longer splice plates to achieve the same amount of fixation obtained if nails were spaced in a single continuous group.

E. George Stern, USA

3567. Stern, E. G., Deterioration of green wood along steel-nail shank and its influence on the nail-holding properties, *Virginia J. Sci.* **1,** 3, 200–218, July 1950.

Green or partially air-dry wood, in long-time contact with steel nails before or during air-seasoning, may deteriorate along the nail shank without any visual evidence of deterioration at the wood surface under normal conditions of air-seasoning. This deterioration may be considerably more detrimental to the holding properties of plain-shank than to those of the helically grooved Screwtite and the annularly grooved Stronghold nails which have been subjected to extensive tests.

Proper surface coating of the steel-nail shank in long-time contact with green or partially air-dry wood may prevent or decrease wood deterioration. Thus, the long-time properties of steel nails driven into green wood may be improved.

From author's summary

3568. Wittwar-Passail, P., Rectangular and round wood beams (in German), *Inter. Holzmarkt* **42,** 9, 14–18, May 1951.

Permissible single and unit loads are presented in both graphical and tabular forms for rectangular and round wood cross sections to allow direct comparisons of their economies.

E. George Stern, USA

3569. Keylwerth, R., The anisotropic elasticity of wood and of laminated wood (in German), *VDI-Forschungsheft* **430,** 40 pp., 1951.

The report consists of four parts: (A) Fundamentals of theory of elasticity. (B) Basic stress states. (C) Determination of elastic constants for wood. (D) Elasticity theory of laminated wood.

(A) The three-dimensional states of stress and of strain, and Hooke's law for isotropic materials are treated. Wood as a rhombic crystalline material, together with the elastic constants of a rhombic system are also discussed.

(B) Pure tension, compression, shear, bending, and torsion are discussed for anisotropic materials. For various orientation of the coordinate axes, the elastic properties of the (anisotropic) wood, the lateral dilatation coefficients, the moduli of elasticity, the inverse shear moduli, and the elastic constants for torsional deformation are given in graphical and in tabular form. The above states of deformation are also analyzed.

(C) A discussion and a table of the elastic constants for various kinds of wood are given.

(D) The relation of mean stress and mean strain for laminated wood (all laminas with parallel grain) are discussed. The influence of synthetic resin-film bond on the modulus of elasticity and on the shear modulus is treated for wood laminates with parallel grain. Finally, wood laminates with veneer grain placed at varying angles are analyzed. For this type of veneer arrangement and for various kinds of wood, various veneer thickness, and various numbers of laminates, the modulus of elasticity, shear modulus, and other elastic constants are given.

Paul Torda, USA

3570. Hopkins, I. L., Dynamic shear properties of rubberlike polymers, *Trans. Amer. Soc. mech. Engrs.* **73,** 2, 195–203, Feb. 1951.

A simple apparatus is described for determining the dynamic shear modulus and damping (dynamic viscosity) of elastomers over the frequency range from 100 to 5250 cps. The specimens are vibrated in shear by an electrically driven tuning fork. Data are given on 10 rubbers and soft plastic materials at 80 F and 150 F. Some scattering due to poor adhesion of the specimens to the tuning fork and other errors is evident. The data show an increase in modulus and a decrease in viscosity as the frequency is increased. The Q value (proportional to the ratio of maximum stored energy to energy dissipated per cycle) does not, in general, change radically with frequency. Although the precision of the method is not high, a large frequency range may be covered, and very small specimens are used.

Lawrence Nielsen, USA

3571. Hsiao, C. C., and Sauer, J. A., Effect of strain rate on the tensile and compressive stress-strain properties of polystyrene, *Amer. Soc. Test. Mat. Bull.* no. 172, 29–35, Feb. 1951.

If stress is a function only of instantaneous values of plastic strain, strain rate and temperature, then tensile creep properties can be derived from simple tension behavior under various constant strain rates. A testing machine was adapted so that strain rate could be controlled by an electric strain gage fixed to specimen.

Experiments show that tensile strength of polystyrene increases with strain rate, fracture strain remaining constant, this being a characteristic feature of brittle fracture. The stress-strain relation is linear in tension but in compression reaches a maximum. Failure in compression occurs by localized bulging of specimen. Tensile creep properties cannot be derived from these tests, however, because the material is brittle and plastic flow too small. Reviewer finds plastic flow very small for polystyrene,

and crazing influences its behavior [*Brit. J. appl. Phys.* **1**, 294–301, 1950]. W. Lethersich, England

3572. Bestul, A. B., Decker, G. E., and White, H. S., Comparison of viscosities of rubbers from the McKee worker-consistometer and from the Mooney viscometer, *J. Res. nat. Bur. Stands.* **46**, 4, 283–287, Apr. 1951.

Viscosities of three GR-S rubbers and three GR-I rubbers were measured at 100 C with the McKee worker consistometer, a piston-type capillary viscometer, and with the standard Mooney viscometer, a rotating disk instrument. Rates of shear from several tenths to several hundred sec^{-1} were covered with the consistometer, and from 0.1 sec^{-1} to several sec^{-1} with the Mooney viscometer. The procedure for preparing the samples and taking the measurements with the Mooney viscometer was selected as giving the best possible simulation of the conditions of the standard Mooney measurement, consistent with a valid indication of the temperature of the sample and as nearly as possible a like state of degradation at all rates of shear. The viscosities from the consistometer were calculated, using the Weissenberg-Rabinowitsch differentiation method for reducing observed flow data to fundamental quantities. The agreement of results from the two instruments in their common range varies for the different rubbers, being better for the GR-S's. The best agreement is within about 2%, the uncertainty of the measurements, and the worst is within about 20%. Results from both instruments for X-518 GR-S agree with results obtained by Treloar for à "standard GR-S" with a Piper and Scott biconical rotor modification of the Mooney viscometer. As the disagreements shown are not the same for all the rubbers, they cannot be the result of constant instrumental differences, but must involve factors such as hydrostatic pressure, elasticity, and slippage at instrument surfaces.

From authors' summary

3573. Kosten, C. W., and Bronkhorst, J. A. J., Vibration dampers (in Dutch), *Tech. Wet. Tijdschr.* **20**, 2, 28–33, Feb. 1951.

Article deals with the elastic behavior of rubber, sketches theoretical research on the subject, and gives a few general results of experimental research. Actual measurements have led to the simple formulas and graphs which will help to calculate cushion foundations on rubber with a fair grade of accuracy. A nomogram has been devised for the same purpose.

Comparisons are given between dampers in rubber, cork, felt, and metal, by considering, among other things, aftereffects, internal friction of the material, and proper vibrations of the cushion. From authors' summary

3574. Amelinckx, S., An interferometric study of cleavage surfaces of artificially grown crystals of sodium chloride, *Phil. Mag.* (7), **42**, 327, 324–330, Apr. 1951.

Cleavage surfaces of an artificially grown crystal of sodium chloride are examined by multiple-beam interferometry. Two main features are noted, namely, long narrow, more or less parallel strips, and superposed on this a secondary fine grain structure. Silver films on sodium chloride are found to be unstable and apparently photosensitive. The cleavage mechanism is discussed. From author's summary

3575. Sioiri, Z., The dynamic elastic properties of twines (in Japanese), *J. mech. Lab. Tokyo* **4**, 2, 41–46, Mar. 1950.

A testing machine was made to investigate the dynamic elastic properties of twines. Testing speed and amplitude of the cyclic elongation by this machine are from $^1/_{60}$ to 3 cps and 0.25%, respectively. The dynamic stress-strain curves of manila and cotton twines were photographed at both dry and wet conditions, and the hysteresis loops of these curves investigated.

The elongation of twines is calculated on the assumption that when the twines are stretched, the yarns press each other to lateral direction, the cross area of twines becomes small, and twines are lengthened without the elongation of yarns. This calculation agrees with results of dynamic and static tests.

From author's summary

3576. Kaufmann, F., Effect of temperature and humidity of air on the behaviour of facing plaster (in French), *Rev. Matér Constr.* no. 423, 370–372, Dec. 1950.

This is a French translation of a German article which appeared in "Zement, Kalk, Gips" for August, 1949. Strangely, it does not give the chemical composition of the plaster which is the subject of the article, but study of a foreign-language dictionary of the building trades indicates that the subject is plaster of Paris (hydrate of $CaSO_4$) and not lime plaster. Most notable among the conclusions is that there is a sharp increase in tensile and compressive strengths of setting plaster as it approaches a free-water content of zero. Results of many tests are given in the article, which is well described by its title.

W. C. Johnson, USA

3577. Tabor, D., The hardness and strength of metals, *J. Inst. Metals* **79**, part I, 1–18, 1951.

Starting from the known relation that for heavily cold-worked metals, which may be regarded as ideally plastic, the ratio of the Meier hardness H_m and yield stress Y (or ult. tens. str.) is constant ($\sim$2.8), author assumes that for metals which strain-harden during deformation, H_m is also 2.8 $\times$ a representative yield stress Y_r of the metal in the deformed zone of the indentation. By taking the representative strain ϵ_r equal to $20\,d/D$ (d is diam impression, D diam ball), the $Y_r - \epsilon_r$ curve quantitatively agrees with the $Y - \epsilon$ curve in compression. Ball hardness tests at different loads therefore enable a determination of the $Y - \epsilon$ curve. Assuming Y a power function of ϵ, it follows that H_m is a power function of d/D (Meier's law) with the same index x. The ratio of T_u and H_m or H_b (Brinell hardness) can then be calculated for strain-hardening metals at different ratios of d/D. By plotting T_u/H_b as a function of x, the calculated and observed values show reasonable agreement for $d/D = 0.7$. T_u/H_b is fairly constant, 0.34 for x between 0 and 0.2 (certain steels and cold-worked metals), but increases with higher values of x up to 0.5 for x = 0.5.

A similar relation is obtained for the Vickers hardness for which ϵ_r is constant (8%) due to geometrical similarity of the impressions. Author admits the limited value of this method due to the invalidity of the power relation between Y and ϵ at higher stresses. In reviewer's opinion, theoretically more correct results will probably be obtained by taking ϵ_r expressed in true strains proportional to d/D (ball) or constant (cone, pyramid) and using the exponential $Y - \epsilon$ relation [*Appl. sci. Res.* Al, 198, 1948; *J. Metals* **185**, p. 185, 1949, *Metalen*, 5, p. 9, 1950] which leads to finite Y at the onset of plastic deformation (initial Y, Hertz hardness) and at infinite deformation (assympt. Y, hardness).

J. H. Palm, Holland

3578. Combes, L. S., Ballard, S. S., and McCarthy, K. A., Mechanical and thermal properties of certain optical crystalline materials, *J. opt. Soc. Amer.* **41**, 4, 215–222, Apr. 1951.

Measurements have been made of various physical properties of certain optical materials of interest in infrared instrumentation. Tables are included which list values of Young's modulus, apparent elastic limit, modulus of rupture, modulus of rigidity,

hardness, and linear coefficient of thermal expansion. Major data are reported on the following crystals: Silver chloride, thallium bromide-iodide, thallium bromide-chloride, sodium chloride, potassium chloride, potassium bromide, lithium fluoride, and calcium fluoride. Some data are also given for a dozen odd other optical materials. Literature values of elastic coefficients are stated for comparison, as well as the values of Young's modulus and modulus of rigidity calculated from these coefficients. A brief discussion is given of the variation of the elastic moduli with direction in the crystal for cubic crystals.

From authors' summary

3579. Jepson, M. D., The effect of isothermal treatment on the mechanical properties of steel, *Metallurgia, Manchr.* **43,** 256, 62–64, 81–82, Feb. 1951.

Comparison is made between isothermally transformed material (tempered in some cases) and quenched and tempered specimens. Results show the isothermal treatment to have an advantage, but the marked inferiority exhibited by the quenched samples tempered at temperatures below 300 C was found to be due to embrittlement by the molten solder of the tempering bath.

From author's summary

3580. Shartsis, L., and Spinner, S., Viscosity and density of molten optical glasses, *J. Res. nat. Bur. Stands.* **46,** 3, 176–194, Mar. 1951.

A counterbalanced sphere method was used to measure the viscosities and densities of 22 molten optical glasses at various temperatures. Six different types of glass were represented. The densities agreed with those obtained by a volumeter method, and the viscosities agreed with those obtained with a small rotation viscometer. Details of the calibration and measuring techniques are given. Authors consider the sphere apparatus to be simple and rugged.

Expansivities in the liquid range (1000 to 1200 C) calculated from density data were from 2 to 3.5 times higher than corresponding expansivities in the solid range (100 to 400 C). Of the glass types, the flints and barium flints had the lowest expansivities, the barium crowns were intermediate, and the silicate crowns had the highest expansivities. A general correlation was found between expansivity in the liquid range and the slope of the viscosity-temperature curve.

A numerical index calculated from viscosity data is suggested for evaluating the molding qualities of a glass. This index was found to correlate well with estimates made by molding supervisors.

W. H. Duckworth, USA

3581. Ailrod, B. M., and Sherman, M. A., Strength of heat-resistant laminates up to 375° C, *Nat. adv. Comm. Aero. tech. Note* 2266, 57 pp., Feb. 1951.

Laminates of glass cloth plus silicone, phenol-formaldehyde, melamine-formaldehyde, acrylic, and unsaturated polyester resins are considered for use in high-speed aircraft in which high temperatures may be encountered. Strength properties at high temperatures and at ordinary temperatures after exposure to high temperatures are therefore important. Samples of material obtained from manufacturers were tested in flexure, after heating at 25 to 375 C for 0.5 to 200 hr. Comparisons are made with heat-deterioration measurements carried out by others. Silicone laminates were best in retention of strength at 250 to 300 C. Tested at 325 C after 0.5 and 200 hr exposure at 250 to 300 C, 30% of flexural strength and 50% of modulus of elasticity were retained. Phenolic laminate tested at 375 C after 0.5 hr exposure retained high strength values, but after prolonged heat of 250 C strength decreased to a small fraction. Melamines were poorer than phenolics after short exposures but better after prolonged heating. Polyesters lost 80% when tested at 250 C, acrylics lost 80% when tested at 150 C. Decreases in moduli of elasticity were in proportion. Paper provides some needed information respecting high-temperature performance of laminates.

Albert G. H. Dietz, USA

3582. Walmsley, J. L., Spherical shells subjected to internal pressure and low temperatures, *Weld. Res. Suppl.* **16,** 3, 153s–167s, Mar. 1951.

Investigation discusses experiments on spherical shells subjected to internal pressure and low temperatures, details of which are covered in a paper by Marin and co-workers [see AMR **3,** Rev. 1489]. Author includes other interpretations than those given in the original paper and concludes: (1) That about 60% of the normal capacity for deformation and energy absorption is retained at −40 F under biaxial tensile stresses, as compared to simple tension at room temperature; (2) that transition temperature for the biaxial state of stress lies considerably below that of the usual type of notched bar specimens; (3) that a notch produced by defective workmanship can be a source for initiating a very brittle type of failure in a biaxially stressed structure at low temperature.

Joseph Marin, USA

Mechanics of Forming and Cutting

3583. Henriksen, E. K., Stress distribution in the continuous chip—a solution of the paradox of chip curl, *Trans. Amer. Soc. mech. Engrs.* **73,** 4, 461–466, May 1951.

Paper is an analysis of the stress distribution in a continuous chip such as occurs in turning ductile metals. From observations and mathematical treatment, an explanation is derived for the curling of the chip. The chip, as it forms, shows a tendency to bend away from the tool behind the "plane of shear." Stresses due to deformation already exist in the chip; additional stresses will be induced by sliding contact between the chip and the tool face, which occurs under high pressures.

Two different forces are at work between the chip and the tool face, a normal pressure and a frictional force, the latter bending the chip toward the tool.

The physical nature of the bending away of the chip from the tool is explained by the inevitable surface roughness of the tool which forces the chip to curl away at some particular distance from the cutting edge.

A. O. Schmidt, USA

3584. Pupke, H., Remarks on the basic equations of the mechanical phenomena in the spinning process of the draw-spinning process of artificial copper-silk (in German), *Kolloid Z.* 118, 1, 33–37, July 1950.

3585. Arai, Ch., Study of the delation of metal in free forging, *Memo. Fac. Sci. Engng. Waseda Univ. Tokyo,* no. 14, 19–21, 1950.

The results of a group of experiments are described, in which rectangular and cylindrical, aluminum alloy, and steel bars are deformed by the forging operation. Relationships between final and initial dimensions are depicted.

Bernard W. Shaffer, USA

3586. Kostron, H., The local strength of extruded profiles, I, II (in German), *Metall* **4, 5;** 21/22, 3/4; 451–458, 58–63; Nov. 1950, Feb. 1951.

Experiments were conducted on extruded "bondur" (94 Al, 4 Cu, 1 Mg, 1 Mn). Hard and brittle grains of intermetallic compounds are formed in rows, the orientation of which depends on

the local flow direction. Small testing rods were cut from the samples parallel to and perpendicular to the rows. The latter show a higher yield value and smaller elongation, but almost the same tensile strength as the former. Of more importance is the difference in endurance limit under alternating stresses.

C. Zwikker, Netherlands

3587. Leyensetter, W., Measurement of chip deformation in turning (in German), *Z. Ver. dtsch. Ing.* **93,** 13, 375–378, May 1951.

The relationship λ = (distance of tool travel)/(actual length of chip) is determined for various rake angles on tools of HSS and carbide when turning mild steel at cutting speeds between 50 and 500 fpm. Compression of the chips is less when carbide tools are used for otherwise identical conditions. The amount of deformation in the chip decreases because of tool wear on HSS tools. Maximum deformation occurs usually at cutting speed between 100 and 200 fpm for HSS tools. Increased compression, which is less pronounced with carbide tools than with HSS tools, is due to welding action at the cutting edge. Negative rake angles cause more deformation than positive rake angles, but higher cutting speeds and carbide tools would decrease the resultant thickness of the chip. A. O. Schmidt, USA

Hydraulics; Cavitation; Transport

(See also Revs. 3685, 3731)

3588. Tison, L. J., Course in hydraulics. Part I. Motion of liquids in conduits. Flow through orifices. Weirs [Cours d'hydraulique. I^re partie. Mouvement des liquides dans les conduits. Ecoulement par les orifices. Déversoirs], Belgium, Gand, 1950, 383 pp.

This is a typical hydraulics textbook as used in most European schools. Contains fundamental equations of fluid dynamics for perfect and viscous fluids and their application to hydrostatic pressure, stability of flotation, flow problems, and similitude. Extensive studies of flow through orifices and over weirs. Laminar and turbulent flow in pipe lines, including water-distribution systems and pipes with multiple discharges. Methods of flow measurement. Excellent, though partial treatment of subject.

André L. Jorissen, USA

3589. Tonini, D., On the possibility of an analytic solution of the trial method of calculation of dams (in Italian), *Energia elett.* **27,** 9, 533–538, Sept. 1950.

Author assumes that the trial load method for arch dam calculation is known. His aim is to develop the calculations in such a way as to make it possible for the equations of elasticity to be solved directly, without any trial calculation.

The calculation is developed in three parts. In the first part, the author calculates the deflections of the cantilevers; in the second, the deflections of the arch which, in the third and final part, have to be equated to the former. The whole calculation is built up on a series of coefficients which give the deflections of the cantilever and of the arch for elemental loads which have to be calculated exactly as if it were for the trial load method, but the coefficients have to be grouped in order to make a direct calculation possible.

Author mentions a similar method developed by Yurecka in an unpublished thesis submitted to the Institute of Technology in Graz. M. Ritter [Zürich Institute of Technology] developed a similar but easier method [see Jaeger, Ch., "Modern trends in arch dam constructions and design," *Civil Engng. Lond.* **45,** nos. 526–531, 1950] which was later generalized by E. Bosshard [Thesis, 1950, Zürich Institute of Technology].

Charles Jaeger, England

3590. Ward, and A. F. H., Neale, S. M., and Bilton, N. F., Viscosity of lubricating oils at high rates of shear, *Brit. J. appl. Phys. Suppl.* **no. 1,** 12–18, Jan. 1951.

Authors used high-speed rotational viscometer with small clearances to obtain rates of shear up to 4×10^{-5} reciprocal seconds. Special instrumentation and cooling method made possible accurate estimates of film-temperature limits. Both straight mineral oils and a mineral oil with varying percentages of viscosity-index improver added were tested.

Viscosity showed a decrease at high shear rates. Fall in viscosity increased with increasing shear rate and decreased with increasing film temperature. Decrease in viscosity was found to increase with the percentage of viscosity-index improver added.

Results are thought to be explainable in terms of opposite effects of orientation caused by velocity gradient and disorientation caused by Brownian motion. William J. Anderson, USA

3591. Patočka, C., Measurement of discharge by means of contracting water streams (in Czech), *Czech. Techn. Univ. Praha,* no. 27, 24 pp., 1950.

Author reviews standard orifices, nozzles, and Venturi meters with formulas for rate of flow. In diagrams, the discharge coefficients are plotted against *Re* number and contraction ratio. Also, corrections are given graphically on effects of roughness. Venturi flumes for open channels (Parshall and Sajenko) are discussed, especially for the case of critical speed in the narrowest section. Otakar Mastovsky, Czechoslovakia

Incompressible Flow: Laminar; Viscous

(See also Revs. 3645, 3662, 3668, 3672, 3701)

3592. Truesdell, C., On the balance between deformation and rotation in the motion of a continuous medium, *J. Wash. Acad. Sci.* **40,** 313–317, 1950.

This paper is a purely kinematic study of fluid motion with emphasis on two scalars: the magnitude w of the vorticity vector $w_i = -e_{ijk}v_{j,k}$, where v_j is the velocity, and an invariant of deformation $\mathrm{II}_d = \frac{1}{2}e_{ijk}e_{ilm}d_{jl}d_{km} = \frac{1}{2}(d_{jj}d_{mm} - d_{jm}d_{jm})$, where e is the permutation symbol and $2d_{jl} = v_{j,l} + v_{l,j}$. Author shows that if the container is fixed and the fluid adheres to it (with suitable conditions at infinity), then there is a balance between vorticity and deformation in the sense that $\int(4\mathrm{II}_d + w^2)dV = 0$. He also discusses motions for which the balance holds at each point ($4\mathrm{II}_d + w^2 = 0$) and generalizes a theorem of Bilimovitch [*Acad. Serbe Sci. Publ. Inst. Math.* 2, 37–52, 1948]. This involves consideration of the velocital volume V_V and the hodograph volume of V_H of an arbitrary region V bounded by a surface S: V_V is the volume enclosed by the surface formed by the extremities of the velocity vectors drawn from the points of S, and V_H the volume formed by drawing these velocity vectors from a fixed point. The dimensional heterogeneity involved in the definitions of these volumes is noted and used.

J. L. Synge, USA

3593. Hofsommer, D. J., Systematic representation of aerodynamic coefficients of an oscillating aerofoil in two-dimensional incompressible flow, *Nat. LuchtLab. Amsterdam Rap.* F.61, 27 pp., 1950.

Problem of thin airfoil is worked out using acceleration potential (linearized pressure) and linearizing boundary conditions.

Splitting the acceleration potential into regular and nonregular parts and using elliptic coordinates gives the solution in a useful form, which can be reduced to usual results. Forces and moments on an airfoil and control surfaces are worked out for translation and rotation of the airfoil, rotation of aileron and tab. A systematic representation of aerodynamic coefficients in terms of tensors is given and symmetries are discussed. Theory of gap is refined by taking account of thickness. Tabulation of numerical values of useful functions is included. Julian D. Cole, USA

3594. **Dörr, J., Exact solution of the integral equation for the flow through a vertical airfoil cascade** (in German), *Ing.-Arch.* **19**, 1, 66–68, 1951.

Author considers congruent airfoils under usual linearizing assumptions of Birnbaum theory. The integral equation for chord vorticity distribution for given downwash, in zero stagger case, is reduced to one with the Cauchy kernel by even-odd split of functions and obvious variable change. Via resolvent kernel due (in aerodynamic contexts) to Betz [cf. Söhngen, H., *Math. Z.* 45, 1939], solution is obtained in closed form analytic in complex separation-stagger parameter, hence applies also for arbitrary stagger and positive separation, or zero separation and overlapping stagger. A. Charnes, USA

3595. **Woolard, H. W., The incompressible flow about a cascade of airfoils,** *Cornell aero. Lab., Inc., Rep.* AF-734-A-1, 40 pp., Nov. 1950.

Conformal transformation methods of calculating velocity distribution about airfoils in cascade have been developed and are now well known, but often require excessive time. On the basis of certain physical and mathematical simplifications, paper develops method requiring much less time for direct problem (calculation of velocity distribution for given airfoil), though probably saving little time for inverse problem (calculation of airfoil shape to obtain given velocity distribution). Comparison with values calculated by conformal transformation methods shows good agreement when blade angle is between 30° and 150°, camber and thickness are not too great, and chord-pitch ratios are unity or less. Method probably is most useful in obtaining quick survey of velocity distribution for given airfoil shape at various angles of attack. C. W. Smith, USA

3596. **Drescher, H., An experimental investigation of aerodynamic forces on an aerofoil with oscillating control surface** (in German), *Öst. Ing.-Arch.* **4**, 3–4, 270–290, 1950.

Unsteady aerodynamic forces acting on fixed airfoil with harmonically oscillating control surface were determined in a water-flow channel by integration of recorded chordwise pressure distribution. Instantaneous pressure was measured at 21 points of mid-span chord with aid of multimanometer with optical registration. Span of airfoil was equal to tunnel width, making flow two-dimensional. Reynolds number, based on chord, varied between 0.3×10^6 and 0.9×10^6. Reduced frequency, based on half chord, was between 0 and 5.0.

Wake vortex pattern, made visible by colored liquids, was photographed. It appears that vortex plane is more deformed for higher reduced frequency.

Experimental results are compared amply with theory. In general, agreement is satisfactory, though, like in steady case, measured lift has smaller amplitude than is predicted by theory. Experiments also show greater retardation in phase. Both phenomena are ascribed to boundary-layer effects. Experimental values for P- or C-function are deduced, which show differences of 10% in magnitude with theoretical C-function.

[Reviewer's note: Position of laminar-turbulent transition point in steady flow is not mentioned; according to measurements made at National Aeronautical Research Institute, Amsterdam (to be published shortly), this position may, in some cases, have appreciable influence on results.] A. I. van de Vooren, Holland

3597. **Yatseev, V. I., On a class of exact solutions of the equations of motion of a viscous fluid** (in Russian), *Zh. eksp. teor. Fiz.* **20**, 1031–1034, 1950.

Author finds an exact solution of the Navier-Stokes equations in the form $v_r = F(\theta)/r$, $v_\theta = f(\theta)/r$, $v_\varphi = 0$, $p/\rho = g(\theta)/r^2$. The solution is given by $F = -f' - f\cot\theta$, $g = -2\nu f' + 2\nu^2(b\cos\theta - a)/\sin^2\theta$, $f = -2\nu\chi'(\theta)/\chi(\theta)$, where

$$\chi(\theta) = (\cos\tfrac{1}{2}\theta)^\gamma (\sin\tfrac{1}{2}\theta)^{1+\alpha+\beta-\gamma}\{c_1 F(\alpha, \beta, \gamma, \cos^2\tfrac{1}{2}\theta) + c_2 F(\alpha+1-\gamma, \beta+1-\gamma, 2-\gamma, \cos^2\tfrac{1}{2}\theta)\},$$

and where α, β, γ are related to the constants a, b, c by the relations

$$a = \gamma^2 - (1+\alpha+\beta)\gamma + \tfrac{1}{2}(\alpha+\beta)^2 - \tfrac{1}{2},$$
$$b = (\alpha+\beta-1)\gamma - \tfrac{1}{2}(\alpha+\beta) + \tfrac{1}{2},$$
$$c = \tfrac{1}{2}[(\alpha-\beta)^2 - 1].$$

The solutions for the case $a = b = c$ are discussed in some detail. J. V. Wehausen, USA

3598. **Carry, C., and Chapus, E., Diffraction of water waves on the lee side of a jetty** (in French), *Houille blanche* **6**, 1, 59–64, Jan.-Feb. 1951.

Starting from the analogy between the displacement vector of surface waves in shallow water of constant depth and the magnetic vector of optical wave motion, the diffraction of surface waves by an infinite perfectly reflecting screen is calculated by Sommerfeld's formulas. Charts are prepared, giving the amplitude for different angles of incidence in various directions. Moreover, asymptotic formulas are given for use at large distances from the edge of the screen. Comparison with measurements shows a good agreement in the regions considered. It is not quite obvious that the boundary conditions on the screen are the same as in the case referred to in the theory, but this apparently does not influence the coincidence of calculated and measured values in the regions considered. R. Timman, Holland

3599. **Morgan, G. W., A study of motions in a rotating liquid,** *Proc. roy. Soc. Lond. Ser. A*, **206**, 1084, 108–130, Mar. 1951.

Motion of solids in fluid rotating with uniform angular velocity (like solid body) is an old problem, attractive from both mathematical and engineering standpoints, which, up to date, has not been solved satisfactorily. A series of papers previously published by G. I. Taylor, J. Proudman, S. F. Grace, and H. Görtler, with experiments by Taylor, did not solve it; they showed only its complicated structure. Following previous writers, author simplifies equations of motion of perfect incompressible fluid by means of linearization (small motion) and axial symmetry and finds the simplest possible solutions. These are: Forced oscillations of rotating fluid in three cases (elliptic, hyperbolic, parabolic); forced oscillations of circular disk with its plane perpendicular to the axis of rotation (disk is replaced by diaphragm which expands and contracts periodically in z-direction); and forced oscillations of the vertical velocity component along the vertical axis (hyperbolic case). Attempt is made to explain how slow, two-dimensional motion can be produced by introducing three-dimensional boundary conditions (as observed experimentally by Taylor) by considering problem from the moment at which disturbance is created from rest relative to the rotating system. Flow is found to become steady and two-dimensional if the disturbance ap-

proaches a steady state (as pointed out previously by Taylor). If the disturbance is due to a body moving along the axis of rotation, steady two-dimensional behavior is everywhere except in the neighborhood of the surface of infinite cylinder enclosing the body. To resolve apparent disagreement between results of Grace and experiments of Taylor, author advances new interpretation of Grace's conclusions. Reader appreciates author's final remarks that his results (based upon Laplace's transforms and Bessel's functions) are conjectures rather than definite conclusions and should be verified by experiments, but may be willing to put down some remarks. Author follows old pattern, i.e., presumes that linearized equations suffice to describe physical phenomena. Perhaps a new pattern, like qualitative discussion of the order of neglected terms, viscous terms (slow motion, the order of these terms may be comparable to terms preserved), etc., may be very (or even more) helpful.

M. Z. Krzywoblocki, USA

3600. Jones, E. E., The effect of the nonuniformity of the stream on the aerodynamic characteristics of a moving aerofoil, *Quart. J. Mech. appl. Math.* **4,** part 1, 64–77, Mar. 1951.

Author considers arbitrary motion of an airfoil in a nonuniform stream whose complex potential is, for practical purposes, given by a fourth-order polynomial. Paper is based on a generalization of the analysis developed by Miss R. M. Morris for the arbitrary motion of an airfoil in a uniform stream [*Proc. roy. Soc. Lond.*, **161, 164, 172, 188**]. For the steady case, results of present paper agree with those given by S. Goldstein [*Rep. Mem. A.R.C.* **1902**] following earlies work by G. I. Taylor. More generally, analysis is designed to include effect of trailing vortex sheets, but this leads to expressions which would be rather difficult to cope with in concrete cases.

A. Robinson, England

3601. Spencer, R. S., and Wiley, R. M., The mixing of very viscous liquids, *J. Colloid Sci.* **6,** 2, 133–145, Apr. 1951.

The process of mixing very viscous fluids such as rubber, dough, or taffy is described and analyzed. Then a method is described of measuring the degree of mixing. This was done by determining the amount of interfacial surface developed during the mixing process and its distribution over the entire volume. Both expressions, amount and distribution, are described mathematically and are used to discuss the most common mixing methods and devices. The entire paper is rather theoretical and does not quote any particular measurements or experiments to support the theory.

H. A. Einstein, USA

3602. Dienemann, W., Calculation of the thermal boundary layer of a body in incompressible laminar flow, *J. aero. Sci.* **18,** 1, 64–65, Jan. 1951.

An approximate method of determining the steady thermal boundary-layer characteristics about a body of arbitrary shape and wall-temperature distribution is briefly outlined. A reasonable approach to the general problems of thermal boundary layers is presented by application of the Holstein-Bohlen version of the Pohlhausen method of determining velocity boundary-layer characteristics. The results of this method are shown to be in excellent agreement with a certain exact solution presented previously by the reviewer.

Leonard Goland, USA

3603. Sowerby, L., The unsteady flow of viscous incompressible fluid inside an infinite channel, *Phil. Mag.* (7), **42,** 325, 176–187, Feb. 1951.

An infinitely long wedge-shaped channel whose walls are two semi-infinite planes inclined at an angle π/n (n is an integer ≥ 1) is suddenly started from rest with constant velocity in the direction of its length (parallel to the z-axis). The search for a solution, for which the only nonvanishing velocity component (w) is the one in the direction of motion of the channel, leads to a linear differential equation (the heat equation) in w, in which w is independent of z. The solution is obtained by means of a suitable Green's function.

An expression for the skin friction is obtained. Its value for $n > 1$ is zero at the corner and is everywhere less than the skin friction for Rayleigh's problem of the infinite plate suddenly started from rest ($n = 1$), but it approaches Rayleigh's value with increasing distance from the corner.

Following Rayleigh, author assumes that his results can be used to obtain a qualitative picture for the steady flow inside a stationary semi-infinite channel having leading edges normal to the incident stream. Particular reference is made to the right-angled channel ($n = 2$) and the results are used to deduce an approximation for the drag on the interior of a finite rectangular duct.

G. Morgan, USA

3604. Etkin, B., and Szebehely, V. G., Comments on Truesdell's paper on Bernoulli's theorem for viscous compressible fluids, *Phys. Rev.* (2) **80,** 4, p. 767, Nov. 1950.

The reviewer [title source (2) **77,** 535–536, 1950] proved the following theorem: Let the curves C be everywhere tangent to the plane of vorticity and velocity and everywhere normal to the curl of the vorticity; then along these curves, in any flow of a viscous incompressible fluid of uniform viscosity, if the vorticity be steady and the extraneous force be conservative, Bernoulli's theorem in the classical form remains valid. Authors discuss a more general result of the same type given by the reviewer [op. cit.]. They find some alternatives, a simpler result gained from stronger assumptions, one redundancy in a special case, an absence of applications, and three misprints.

C. Truesdell, USA

3605. Heidmann, M. F., and Humphrey, J. C., Fluctuations in a spray formed by two impinging jets, *Nat. adv. Comm. Aero. tech. Note* 2349, 35 pp., Apr. 1951.

Upon impingement of two jets, a ruffled sheet of liquid forms perpendicular to the plane of the two jets. The liquid sheet disintegrates intermittently, forming a group of drops that appear as waves propagating from the point of impingement. The intermittent disintegration of the liquid sheet results in irregular spacing between waves and in variable wave intensity. There is an abundance of small waves, with the number of waves above a given intensity decreasing as the intensity increased.

The frequency of wave formation is constant over a finite time interval under constant operating conditions. The frequency varied between 1000 and 4000 cps for the range of test conditions used in this investigation. An increase in jet velocity results in an increase in wave frequency, the relation approaching a direct proportionality. For the jet diameters and velocities used in this investigation, an increase in jet velocity of 30 fps resulted in an increase in frequency of approximately 2500 cps. An increase in impingement angle results in a decrease in wave frequency for impingement angles of from 50° to 100°. The decrease in frequency with impingement angle is approximated by the decrease in the cosine of one-half the impingement angle.

A diameter change from 0.025 to 0.057 inch has a negligible effect on wave frequency compared to the effect of jet velocity and impingement angle. A change in jet length from 10 to 80 diameters before impingement has a negligible effect on wave frequency.

From the photographic and frequency data obtained, it appears that ruffling of the liquid sheet persists to the point of disintegration and determines the frequency of the wave formation.

and that irregularities in the jets before impingement may be as instrumental in controlling the ruffling of the liquid sheet as is the friction of the air. From authors' summary

3606. **Rubesin, M. W., The effect of an arbitrary surface-temperature variation along a flat plate on the convective heat transfer in an incompressible turbulent boundary layer,** *Nat. adv. Comm. Aero. tech. Note* 2345, 37 pp., Apr. 1951.

Author determines an approximate solution of the energy equation for turbulent boundary layer on a heated flat plate neglecting viscous dissipation, and then extends this solution to apply to an arbitrary surface temperature by employing an integral solution. All physical properties of the fluid remain constant. To obtain the local heat-transfer coefficient, the well-known $^1/_7$-power-law relationships are used and another $^1/_7$-power law is supposed to hold for temperature distribution across the boundary layer. Hereby a parameter m is introduced, value of which is chosen to fit in the result with existing experimental data.

From the investigations it can be concluded that heat-transfer coefficient measurements by a plug-type heat meter sometimes deviate considerably from true values because of the existence of a stepwise-discontinuous surface temperature upstream of the instrument.

The paper is completed by some algebraic equations for convenient application of the results. Julius Rotta, Germany

3607. **Hawthorne, W. R., Secondary circulation in fluid flow,** *Proc. roy. Soc. Lond. Ser. A*, **206**, 1086, 374–387, May 1951.

Author considers steady, rotational flow around a bend of a non-viscous, incompressible fluid with an initially nonuniform velocity distribution. The component of vorticity resolved in the direction of flow results in a secondary circulation. An expression is derived for the change of this secondary circulation along a streamline. It is shown that the secondary circulation remains unchanged if the streamlines are geodesics on Bernoulli surfaces (constant total pressure). An approximate solution for bent circular pipes gives the interesting result that the secondary flow is not spiral, but that the circulation direction changes periodically, giving rise to an oscillatory flow. Comparison with experiments in bent circular and rectangular ducts shows that the theory correctly reproduces the essential features of the secondary flows. As a consequence of this comparison, it is indicated that friction in the bend itself tends to reduce rather than increase the secondary circulation. Phillip Eisenberg, USA

Compressible Flow, Gas Dynamics

(See also Revs. 3641, 3647, 3653, 3667, 3683)

3608. **Ting, L. and Ludloff, H. F., Aerodynamics of blasts,** *J. aero. Sci.* **18**, 2, 143–144, Feb. 1951.

A uniform plane shock wave, traveling parallel to a smooth plane wall, meets a slight bend in it which gives rise to Mach reflection. The disturbance expands nearly steadily with time and the pressure change Δp is treated as a first-order perturbation. Boundary condition at shock front is replaced by one at reflecting surface in such a form that a (Possio) solution for Δp in the form of a double integral is obtained. Solution is more general than Lighthill's [AMR **3**, Rev. 2716] in that the bent part of wall need not be plane. An interesting result is that flow is strongly rotational in triangular region between slipstream and shock front but elsewhere is irrotational to first order.

An example is given for a strong incident shock with supersonic flow behind it. H. H. M. Pike, England

3609. **Liepmann, H. W., Roshko, A., and Dhawan, S., On reflection of shock waves from boundary layers,** *Nat. adv. Comm. Aero. tech. Note* 2334, 100 pp., Apr. 1951.

This useful and informative report gives qualitative and quantitative results for shock-wave and boundary-layer interactions. Experiments were conducted on the reflection of impulse-type, step-type, and conical shock waves from boundary layers as well as shock reflections from a slip-stream or shear layer. Pressure distributions and schlieren photographs are presented for the above interactions and for shock waves produced by a corner.

Techniques for producing and identifying turbulent and laminar boundary layers in a uniform supersonic flow are also given and comparisons with the subsonic cases are made.

The reflection of an oblique shock wave, developed in a supersonic wind tunnel from a flat plate at a Reynolds number of 0.9×10^6, in the Mach number range of about 1.3 to 1.5, results in an upstream influence of about 50 boundary-layer thicknesses for the laminar layers, as compared with about 5 for the turbulent layer. Separation did not occur at the point of interaction for the turbulent case but was found to occur in almost all of the laminar layers. At a distance from the plate the incident and reflected shocks attain the form predicted by the inviscid theory. Models for both types of boundary-layer interactions are presented and their effects discussed in some detail. Similar results also hold for a shock wave produced by a corner.

Elegant instrumentation and tunnel-flow techniques are given, which will be of considerable assistance for future investigations of this type of interaction and for boundary-layer studies in uniform supersonic flow. I. I. Glass, Canada

3610. **von Kármán, T., The theory of shock waves and the second law of thermodynamics** (in Italian), *Termotecnica* **5**, 2, 82–83, Feb. 1951.

Author develops pedagogically interesting treatment of normal shock, showing that expansion shock waves violate second law of thermodynamics in classical sense of impossibility of perpetual motion of second kind, i.e., impossibility of raising temperature of fluid from low temperatures to high temperature level without mechanical work. Pointing out that a shock wave is a sharp change of density, pressure, temperature, and velocity in a flow with mass flow per unit area, impulse and total energy per unit mass remaining unchanged, author derives the parabolic relation between velocity and total energy per unit mass.

Addition of heat energy can accelerate a subsonic flow only to sonic velocity; addition of heat to a supersonic flow decelerates it to sonic velocity. The change from supersonic to subsonic velocity at given energy is considered as addition of heat to decelerate flow to sonic velocity, instead of as a jump, and the subtraction of equal amount of heat to decelerate flow to the lower velocity. This is addition of heat at high temperature and removal at low temperature. Reverse procedure from subsonic to supersonic flow at same energy is shown to be addition of heat at low temperature and removal at high temperature, a process requiring that mechanical work be done. H. Guyford Stever, USA

3611. **Sakurai, A., On the theory of cylindrical shock wave,** *J. phys. Soc. Japan* **4**, 199–202, 1949.

A radial flow of a compressible fluid with constant viscosity and heat-conduction coefficients is considered. The equations of motion are integrated numerically by taking special values of the mass flow and the Prandtl number. The solution admits three different types of flows, namely: (a) The velocity gradient is entirely negative; (b) it is first negative and then positive; and (c) it is positive in the whole flow field. The first type corresponds to flows that are subsonic at infinity and supersonic in the

neighborhood of the origin. The second corresponds to flows that are subsonic at infinity, become supersonic, and then quickly return to subsonic flow. The third type is subsonic near the origin but supersonic away from it. Except for the first type, the physical existence of these flow types is questionable. Furthermore, author's interpretation of the pressures in a Laval nozzle in the light of these results is difficult to follow.

Y. H. Kuo, USA

3612. Mayenfisch, E., Impact of a moving gas column against a stationary one (in German), *Schweiz. Arch.* **17,** 4, 119–126, Apr. 1951.

Author uses Hugoniot relations to treat compression and rarefaction waves arising from impact of moving and stationary ideal gas streams in a pipe. He considers five cases determined by initial values of gas parameters; three are solved graphically.

Carl E. Pearson, USA

3613. Bruniak, R., On an application of the Crocco vortex-law (in German), *Öst. Ing.-Arch.* **4,** 3–4, 325–333, 1950.

Although Crocco's theorem was derived a long time ago it was not widely applied in engineering problems. In a very instructive manner, author shows the possibility of using it as a proper tool in some practical applications. Starting from the known equations of an oblique shock in an inviscous isentropic flow, he calculates gradients of energy and entropy across an oblique shock in Crocco's formula in terms of the shock and deflection angles. Results obtained are applied to a shock curve, enabling one to calculate the terms due to vorticity in Crocco's equation of the stream function in passing through the shock. As a particular example, author assumes a shock curve in form of an arc of a circle and shows the methods of applying the derived formulas. Crocco's terms due to vorticity appear to be a linear function of velocity. The mathematics used in the derivation is simple and does not go beyond elementary vector calculus. Paper represents a nice contribution to shock theory.

M. Z. Krzywoblocki, USA

3614. Davidson, I. M., Some properties of the compression shock as in turbine and compressor blade passages, *Proc. Instn. mech. Engrs.* **161,** *W. E. P.* no. 52, 187–202, 1949.

In the first part of the paper, the isentropic expansion of a compressible fluid and a classical compression-shock theory are described. Author here tries to introduce a classification of various forms of compression shocks. Next, the phenomena associated with shock-wave reflection and boundary-layer interaction are dealt with, and, finally, some examples are given of the occurrence of shocks in practice. Some of these are illustrated with schlieren pictures. The paper is accompanied by extensive discussion and communication.

Erik Petersohn, Sweden

3615. Kestin, J., The influence of the temperature variation of the specific heats of air in shock-wave calculations, *J. aero. Sci.* **18,** 5, 351–353, May 1951.

For an ideal gas with variable specific heat, provided enthalpy h is a nearly linear function of temperature, sufficiently accurate solutions of the Rankine-Hugoniot equations can be obtained from Prandtl's equation $u_0 u_1 = (a^*)^2$. Here u_0 and u_1 are fluid velocities relative to the shock, and the sonic fluid velocity a^* is given as a graphical function of stagnation enthalpy $(h + \frac{1}{2} u^2)$.

H. H. M. Pike, England

3616. Ovsyannikov, L. V., Gas flow with straight transition line, *Nat. adv. Comm. Aero. tech. Memo.* 1295, 13 pp., May 1951.

See AMR 4, Rev. 778.

3617. Reissner, E., On the application of Mathieu functions in the theory of subsonic compressible flow past oscillating airfoils, *Nat. adv. Comm. Aero. tech. Note* 2363, 42 pp., May 1951.

Report first presents comprehensive account of known results concerning problem stated in title for two-dimensional flow, viz., an explicit solution in terms of Mathieu functions. Author's method for treatment of three-dimensional incompressible case [AMR **1,** Rev. 718], which has already been applied to subsonic compressible case [AMR **3,** Rev. 2724], is adapted to the use of Mathieu functions by showing how the Mathieu-function solution of the two-dimensional problem is affected by the approximative incorporation of three-dimensional effects. Investigation is restricted to wings with rectangular plan form or with moderate sweep. No numerical results are involved.

A. van Heemert, Holland

3618. Broer, L. J. F., On the influence of acoustic relaxation on compressible flow, *Appl. sci. Res. Sec. A,* **2,** 5–6, 447–468, 1951.

Chief object of this paper is to discover perturbations in the motion of a gas produced by heat capacity lag, in contrast to previous work (by reviewer) in which the unperturbed flow is used as a basis for calculation of the entropy increases only.

It is shown that even in an inviscid fluid, vortex can appear due to heat-capacity lag. For example, flow at the nose of an impact tube is shown to shed ring vortexes encircling the tube.

Turning to two-dimensional supersonic flow and one-dimensional unsteady flow, author derives conventionally the equations of the characteristics and shows that these correspond to the propagation of waves with sonic velocity corresponding to the high-frequency (large) speed of sound. However, this higher sonic velocity is only the velocity of propagation of "precursor waves" in the sense of Sommerfeld and Brillouin, and, in the main disturbance which follows, adjustments in the lagging heat capacities are made. In connection with this type of disturbance, author discusses shock waves and finds a minimum shock strength possible. He finds that a shock wave is made up of a discontinuity followed by a region in which the lagging heat capacity is adjusted. Thus the shock waves have a total thickness which depends upon the relaxation time of the gas.

Remainder of paper is devoted to the discussion of "transonic flow," by which author means steady flow at velocities intermediate between the low- and high-frequency sonic speeds. He gives two arguments to show that flow in this velocity range is unstable. The first shows that in *strictly* one-dimensional "transonic" flow a velocity perturbation inevitably involves a velocity gradient, tending to increase this perturbation as we move in the flow-direction. This argument is inconclusive to reviewer, since in a real steady flow a perturbation in velocity would be accompanied by variations in stream-tube area, which would destroy the validity of his analysis which is applicable only to the case of thickened shock waves which are strictly one-dimensional. It means, then, that if there is a velocity perturbation in the "transonic" region of a strictly one-dimensional flow, it must constitute a portion of a shock wave and does not correspond to any real flow which is not strictly one-dimensional.

The second argument shows that entropy increases result inevitably from velocity perturbations in the "transonic" region. The author interprets this as another indication that these velocity perturbations will increase. The reviewer interpret's is that the existence of such entropy increases is merely further evidence that he is in the midst of one of his thickened shock waves.

More generally it can be seen that the statement that uniform steady flow in a certain velocity range is unstable has no meaning since the flow velocity depends upon the coordinate system used

Any such instability must be related to gradients in the unperturbed flow rather than to the flow velocity.

Arthur Kantrowitz, USA

[See author's comment under "Communications" in this issue.]

3619. Lagerstrom, P. A., and Graham, M. E., Methods for calculating the flow in the Trefftz-plane behind supersonic wings, *J. aero. Sci.* **18, 3, 179–190, Mar. 1951.**

By means of the linearized nonviscous theory, methods are developed, using the complex functions of conical flow, for finding downwash and sidewash in the Trefftz-plane behind a supersonic wing. Equations are derived which describe this flow field behind several types of "flat plate" wings: the wide delta wing, the trapezoidal wing (wide delta wing with cutoff tips), and the delta control surfaces (a narrow delta wing, the two sides of which are deflected differentially). Graphs are included which show the results for these cases.

From the authors' summary by J. S. Isenberg, USA

3620. Robinson, A., and Young, A. D., Note on the application of the linearized theory for compressible flow to transonic speeds, *Aero. Res. Counc. Lond. Rep. Mem.* **2399, 6 pp., Jan. 1947, published 1951.**

See AMR **2,** Rev. 217.

3621. Gravalos, F. G., Flow of nonviscous elastic fluids, *J. aero. Sci.* **17, 6, p. 375, June 1950.**

3622. Munakata, Ken-iti, Successive approximation in the hodograph method for treating compressible flow (in Japanese), "Studies in mathematical Physics," Iwanami Shoten Publ. Tokyo 1, 93–129, 1950.

In terms of complex variables $w = u + iv$, $z = x + iy$, $W = \phi + i\psi$, where (u, v) are the velocity components, ϕ the velocity potential, ψ the stream function, the hodograph equation is written in the form

$$\partial f/\partial w = \frac{1}{2}\left(1 - \frac{\rho}{\rho_0}\right)\bar{z} + \frac{1}{4}\frac{\rho}{\rho_0}\left(\frac{c_0}{c}\right)^2 M^2\,(w\bar{w}\bar{z} - \bar{w}^2 z) \quad [*]$$

where $(\rho/\rho_0)^{\gamma-1} = (c/c_0)^2 = 1 - \frac{1}{2}(\gamma - 1)\,M^2 w\bar{w}$, M is the Mach number, and f is the Legendre transform of W such that

$$f = \frac{1}{2}\left(1 - \frac{\rho}{\rho_0}\right) w\bar{z} + \frac{1}{2}\left(1 + \frac{\rho}{\rho_0}\right) \bar{w}z - W,\ z = \partial f/\partial \bar{w} + \partial \bar{f}/\partial w.$$

Assuming $z = \Sigma z_n M^{2n}$, $W = \Sigma W_n M^{2n}$, $f = \Sigma f_n M^{2n}$, substituting them into [*], and equating terms of equal powers in M^2, f_n, z_n, W_n would be determined in succession. The method is applied to the flow past nearly elliptic cylinders (derived from $W_0 = z_0 + z_0^{-1}$), discontinuous flow past a flat plate (Kirchhoff's problem), jet issuing from Borda's mouthpiece, and to the jet from a slit in an infinite plate. In the case of Borda's mouthpiece, exact expression for the coefficient of contraction is obtained by means of momentum consideration. In each case (except the last) various formulas are obtained exact to M^4, while in the last case the result is correct to M^2. Also, critical Mach numbers at which limiting lines first appear are calculated numerically.

Isao Imai, Japan

3623. Moore, F. K., Three-dimensional compressible laminar boundary-layer flow, *Nat. adv. Comm. Aero. tech. Note* **2279, 38 pp., Mar. 1951.**

By using a two-component vector potential, Howarth's transformation, and linear viscosity-temperature relation, the three-dimensional compressible laminar boundary-layer equations are simplified. Mangler's transformation is employed to further simplify equations for application to axially symmetric flow and also to flows for which region exterior to boundary layer is conical. Boundary layer on yawed infinite cylinder is analyzed and it is stated that simple separation of the cross flow from parallel flow (independently discussed previously by Prandtl, Jones and Sears), is not strictly applicable to compressible flow.

By presuming uniqueness and steady flow, author deduces that for flow over a flat plate not having a straight leading edge the boundary-layer equations are inapplicable in a narrow region downstream of any sharp corner of the leading edge.

[Reviewer's note: Reference should be made to the recent work of W. D. Hayes (U.S. N.O.T.S. RRB 105, Nov. 1950) who also has discussed the general three-dimensional compressible boundary-layer equations and their application to several special types of flow.] Dean R. Chapman, USA

3624. Klunker, E. B., and Ivey, H. R., An analysis of supersonic aerodynamic heating with continuous fluid injection, *Nat. adv. Comm. Aero. Rep.* **990, 10 pp., 1950.**

See AMR **3,** Rev. 956.

3625. Valerino, M. F., and Doyle, R. B., Method for determining pressure drop of monatomic gases flowing in turbulent motion through constant-area passages with simultaneous friction and heat addition, *Nat. adv. Comm. Aero. tech. Note* **2328, 37 pp., Apr. 1951.**

Authors state there is interest in the use of monatomic gases as the working medium in various aircraft cooling cycles which are closed and which involve heat exchangers operating at high heat-transfer rates and at comparatively high speeds (high subsonic Mach numbers). Paper presents method of analyzing flow in heat-exchanger passages treated as constant area ducts with friction and heat transfer. Basic one-dimensional, steady-flow equations are arranged in manner found convenient in past. Thus all flow variables are determined once the local impulse and stagnation temperature are known. By use of an empirical relation between friction and heat transfer, an ordinary differential equation relating to the local impulse and the ratio of the local stagnation temperature to wall temperature is established and integrated numerically from initial conditions. A second differential equation giving this same temperature ratio as a function of the distance along the duct is set up and solved. From these solutions all flow variables can be determined along the passage from initial conditions. Charts permitting rapid determination of the pressure drop in the passage under a variety of conditions are given. Paul A. Libby, USA

3626. Reissner, E., Extension of the theory of oscillating airfoils of finite span in subsonic compressible flow, *Nat. adv. Comm. Aero. tech. Note* **2274, 16 pp., Feb. 1951.**

Paper extends author's previous results for two-dimensional oscillating airfoils in subsonic flow [AMR **3,** Rev. 2724] to include finite-span wings. It is shown that the three-dimensional corrections involve only the solution of a one-dimensional integral equation for the spanwise variation of circulation, provided tabulated values of the kernel are known. Max A. Heaslet, USA

3627. Cherry, T. M., Exact solutions for flow of a perfect gas in a two-dimensional Laval nozzle, *Proc. roy Soc. Lond. Ser. A,* **203, 1075, 551–571, Oct. 1950.**

It is known that the solution of an isentropic compressible flow through a convergent-divergent channel, which is subsonic and

supersonic in, respectively, the convergent and divergent sections, becomes three-valued in the supersonic region when mapped into the hodograph plane. To deal with this difficulty, author first expresses the hypergeometric functions in terms of Bessel functions and then transforms part of the solution into Kapteyn series. By the known properties of a summed Kapteyn series, author is able to construct a solution possessing all the desired behaviors. Since this series is singular on the branch lines, the solution is continued to the rest of the region by a number of steps, each one of which is restricted to a specific but overlapping domain. Y. H. Kuo, USA

3628. Stewart, H. J., and Li, T.-Y., Source-superposition method of solution of a periodically oscillating wing at supersonic speeds, *Quart. appl. Math.* **9, 1, 31–45, Apr. 1951.**

In simple concise terms, authors discuss the application of the "equivalent area" theorem first presented by Evvard [AMR **1,** Rev. 1512] to the case of the oscillating airfoil with subsonic edges in a supersonic stream. Results cannot be applied generally to arbitrary nonsteady motions and an example of the breakdown in the theory is given for the case of the unit step function. Correspondence between the theorems developed by the authors with those of previous contributors is given.

James B. Duke, USA

3629. Miles, J. W., The oscillating rectangular airfoil at supersonic speeds, *Quart. appl. Math.* **9, 1, 47–65, Apr. 1951.**

The problem first attacked is that of a rectangular wing tip of no thickness executing a harmonic oscillation whose amplitude is arbitrarily distributed over the wing surface. The linear small-perturbation theory is used, and the technique is that of the Fourier integral in a complex plane. It is easy to write down formally a general solution that reduces to the correct velocity at the wing surface, but some study is required to establish the correct paths of integration.

The solution of the transform equations is carried out with the aid of the Wiener-Hopf technique. Instead of an arbitrary distribution of amplitude, $\alpha(x, y)$ is now assumed to have the form $y^n \exp(-\beta y)1(y)\alpha_n(x)$, where the x-axis lies in the stream direction. (In subsequent applications only the value $\beta = 0$ is used. It is not clear whether the exponential in $\alpha(x, y)$ was included for generality or was required for convergence, since the limiting process $\beta \to 0$ is not discussed.) The final inversion, which would yield the pressure distribution, cannot be carried out explicitly, but information on behavior at small and large y can be deduced. An expansion for small frequency of oscillation k can also be carried out. Certain integrals with respect to y giving chordwise distributions of lift, pitching, and rolling moments, etc., can be evaluated without approximation, for $\alpha(x, y) = y^n\alpha_n(x)$, and are applied specifically to the cases $n = 0$ and, finally $\alpha(x, y) =$ constant, $k = 0$. Some remarks are made about the application to cases of arbitrary time dependence.

Reference is made to numerical results for rectangular wings in plunging and pitching oscillations [AMR **4,** Rev. 847]. It is also stated that whereas the present results are in agreement with Stewartson's [AMR **4,** Rev. 2121], they do not agree with those of Stewart and Li [AMR **4,** Rev. 1267]. W. R. Sears, USA

3630. Li, T.-Y., Purely rolling oscillations of a rectangular wing in supersonic flow, *J. aero. Sci.* **18, 3, 191–198, Mar. 1951.**

This paper may be regarded as a continuation of earlier work by H. J. Stewart and the present author [AMR **4,** Rev. 1267]. Analytical and numerical results are presented for the rolling oscillations of a rectangular wing in supersonic flow. The method is based on an extension of Evvard's principle which is derived elsewhere by H. J. Stewart and the author (see second preceding review). It is found that, for certain frequencies, the damping derivative changes sign with aspect ratio.

It may be noted here that an operational method for calculating the forces on an oscillating rectangular wing in supersonic flow has been developed by J. W. Miles (see preceding review) and numerical results have been worked out but not published. Miles states that for pitching and plunging, his results are at variance with those of H. J. Stewart and the present author in the first paper quoted above. A detailed comparison of the methods employed would appear to be difficult.

A. Robinson, England

3631. Froehlich, J. E., Nonstationary motion of purely supersonic wings, *J. aero. Sci.* **18, 5, 298–310, May 1951.**

The forces and moments on wings can be calculated if certain integrals, taken across the span, of the perturbation potential are known. [This idea was used by Miles (AMR **4,** Rev. 332); also see third preceding review.] Here, the edges being all supersonic, the integrals can be taken over an infinite interval; viz.

$$\psi_1(x, z, t) = -\infty\int^{\infty} \varphi dy \text{ and } \psi_2(x, z, t) = -\infty\int^{\infty} y\varphi dy$$

where φ is the perturbation potential. It is shown that ψ_1 and ψ_2 satisfy the wave equation and standard boundary conditions, and therefore can be calculated by familiar methods of three-dimensional steady supersonic wing theory. It is specified that the trailing edge be straight and perpendicular to the direction of flight. General forms are written down for oscillating wings. The oscillating delta wing is treated in detail for plunging, pitching, and rolling, and for the case of an oscillating flap. The various force and moment coefficients are shown to be independent of sweep angle.

The numerical results are given in terms of certain functions, which also appeared in the two-dimensional oscillating-wing work of Borbely [*Z. angew. Math. Mech.* **22,** 130–205, 1942; RTP trans. no. 2019]. The functions are given here in tables and graphs for Mach numbers of 1.25 and 2.0. Negatively damped pitching oscillations are shown to occur under some conditions; this phenomenon is investigated further. W. R. Sears, USA

3632. Pistolesi, E., On the singularity method in supersonic aerodynamics of aerofoils (in Italian), *Termotecnica* **5, 2, 86–89, Feb. 1951.**

In the linearized supersonic-flow theory the potential of the flow around a wing may be constructed with the help of sources, vortexes, and doublets by an integration process [see Puckett, A., *J. aero. Sci.*, p. 475, 1946]. Author shows with great care all mathematical details proving that the potential function thus produced satisfies the flow equation and that its derivatives with respect to the space coordinates have the qualities expected at the wing surface. I. Flügge-Lotz, USA

3633. Costello, G. R., Cummings, R. L., and Sinnette, J. T., Jr., Detailed computational procedure for design of cascade blades with prescribed velocity distributions in compressible potential flows, *Nat. adv. Comm. Aero. tech. Note* **2281. 49 pp., Feb. 1951.**

A detailed step-by-step computational outline is presented for the design of two-dimensional cascade blades having a prescribed velocity distribution on the blade in a potential flow of the usual compressible fluid. This outline is based on the assumption that the magnitude of the velocity in the flow of the usual compressible fluid is proportional to the magnitude of the velocity in the flow of a compressible nonviscous fluid with linear pressure-volume relation. The computational procedure [following G. R. Costello; see AMR **3,** Rev. 941] includes several ways of adjusting

the prescribed velocity to satisfy restrictions imposed by the method. Tables of coefficients are given for evaluating the necessary integrals, and numerical examples are included.

From authors' summary by E. R. G. Eckert, USA

3634. **Bugaenko, G. A., On the problem of gas flow over an infinite cascade using Chaplygin's approximation,** *Nat. adv. Comm. Aero. tech. Memo.* 1298, 16 pp., May 1951. Translation from Russian.

See AMR **3**, Rev. 1334.

3635. **Krzywoblocki, M. Z., On the limiting lines in inviscid rotational and viscous flows,** *Bull. Calcutta math. Soc.* 42, 3, 153–171, Sept. 1950.

The problem of limiting lines in plane, axially symmetric, and even three-dimensional inviscid flow has been attacked and solved by several authors. Here an attempt is presented to solve the problem of the existence or nonexistence of a limiting line in a viscous fluid. The attack is based on the extended Tollmien's theory. The considerations given below refer both to an inviscid rotational and to a viscous flow. It will be shown that under certain conditions, less restrictive for an inviscid fluid and more restrictive for a viscous fluid, both types of flow may be treated simultaneously. The author does not attempt to present the way of construction of the solution and of the intermediary items (like characteristics, Mach lines, limiting lines, etc.), giving only the proof concerning the existence theorem. This approach in the case of a viscous fluid enables one to use the characteristics of the reduced-order equation (second-order) instead of the singularities of the original equation (third-order). Thus, although one is not able to find the singularities in a viscous-fluid vector field of flow, one may extend the considerations referring to a limiting line to this type of flow. A brief representation of the hypothesis of a shock in viscous gases closes the paper.

From author's summary

3636. **Lukasiewicz, J., Flow in a shock tube of non-uniform cross-section,** *Nat. Res. Counc. Canad. mech. Engng. Rep.* MT-11, 9 pp., Jan. 1950.

A one-dimensional theory for a shock tube has been developed in which the compression and expansion chambers (separated by a diaphragm) have different cross-sectional areas. Depending on the initial conditions, the expansion wave, generated by the rupture of the diaphragm, may or may not follow the contact surface and shock wave into the expansion chamber, and each case must be treated separately. The analysis shows that when the compression compartment has the largest cross section, a shock wave of given strength can be produced at a lower pressure ratio than that in a tube of constant cross section. However, before the theory can be accepted for application to shock-tube design, it appears desirable to investigate experimentally the viscosity effects at the discontinuity in the tube and the extent to which the expansion wave may be regarded as centered and one-dimensional.

Gordon N. Patterson, Canada

3637. **Martin, M. H., Steady, plane, rotational Prandtl-Meyer flow of a polytropic gas,** *J. Math. Phys.* **29,** 4, 263–281, Jan. 1951.

Continuation of study reported in earlier paper [AMR **4,** Rev. 1672] of flows author calls GPM-flows (general Prandtl-Meyer). These are flows having a one-dimensional hodograph and satisfying certain additional restrictions including the finiteness and nonvanishing of the pressure gradient along each streamline. [Reviewer's note: This latter restriction makes the "general" in name "GPM" rather misleading, because it excludes many non-trivial rotational PM flows.] Major result is proof that in isoenergetic GPM-flow of a polytropic gas the velocity vector (in physical plane) is constant on a family of straight lines forming either a parallel or a concurrent array. Some interesting geometrical properties of the hodograph curves are also established. Author then devotes considerable space to integration of the equations and classification and exhibition of flow patterns associated with parallel and concurrent straight line isovels. This latter material in large part duplicates results contained as special cases in earlier papers dealing with "more general than GPM" Prandtl-Meyer flows: [Prim, R., and Neményi, P., AMR **3**, Rev. 2006. Prim, R., AMR **1**, Rev. 83. Prim, R., "Rotational gas flow around a corner," *Sev. Inter. Congr. appl. Mech.*, London, 1948, and *Nav. Ord. Lab. Memo.* 9642, 1948. Prim, R., "Gas flows having a reduced velocity vector of the form $\bar{w} = \bar{\imath}u(y) + jv_1(x)v_2(y)$," *Nav. Ord. Lab. Memo.* 10347, 1950.] R. C. Prim, USA

3638. **Clauser, F. H., Two-dimensional compressible flows having arbitrarily specified pressure distributions for gases with gamma equal to minus one,** *Symp. theor. comp. Flow,* 1949, *Nav. Ord. Lab. Rep.* NOLR-1132, pp. 1–33, 1950.

Mangler [*Jahrbuch Deutsch. Luftfahrtforsch.* **1938,** 46–53] gave a procedure for aerodynamic design by computing the shape of solid surfaces such that the exact potential flow past them should have a prescribed distribution of surface velocity as function of distance along the surface. However, this problem has in general no real solution, and the calculated flow is on a Riemann surface with an infinity of sheets. Hence the reviewer [*Aero. Res. Counc. Lond. Rep. Mem.* 2112, 1945] advanced an improved method in which the velocity distribution was presented on a circle, such that the flow field was imagined transformed into the domain outside it. Then the conditions for existence may be imposed in advance, while the velocity-distance relation may also be checked in advance (for practical utility), with little labor.

Present paper extends this work to general subsonic flow, under the Kármán-Tsien approximation. Thus it enables airfoils to be designed with a given surface-pressure distribution at a given high subsonic speed. Airfoils with slots, slats, and other ducting, and airfoils in cascade, are also treated, following discussions by many authors for the case of zero Mach number. However, there is no explicit statement that the calculation has been carried out for any particular case. In its absence one can only assume that the many illustrations in the text represent conjectural, not calculated, shapes. M. J. Lighthill, England

3639. **Perls, T. A., and Rich, H. L., Evaluation of selected shock instruments,** *David W. Taylor Mod. Basin Rep.* 720, 58 pp., Feb. 1951.

Selected shock instruments have been evaluated in a series of tests in which they were subjected to both steady-state and shock motions. The instruments include velocity meters, accelerometers, multifrequency-reed gages, and ball-crusher gages. It is concluded from these tests that the principle of the multifrequency-reed gage is potentially most useful for the determination of displacement-frequency data needed for the design of shock-proof equipment and structures. However, the present gage does not adequately provide these data except possibly as averages of many data from several gages. In addition, velocity or acceleration data are needed to determine specific shock-excitation frequencies. Both types of meters may introduce their own resonant frequencies. Until a better reed gage can be designed, built, and tested, reliable displacement-frequency data will have to be obtained from torsion-pendulum or analog-computer analyses of velocity-meter and accelerometer records, or by recording an accelerometer output through a set of tuned electrical

circuits. The suitability of the instruments for these purposes is discussed. Use of electrical differentiators and integrators is recommended in some cases. From authors' summary

Turbulence, Boundary Layer, etc.

(See also Revs. 3602, 3606, 3624)

3640. Howarth, L., The boundary layer in three-dimensional flow. Part I. Derivation of the equations for flow along a general curved surface, *Phil. Mag.* (7) **42**, 326, 239–243, Mar. 1951.

For setting up the general equations of boundary-layer flow along a curved surface, a system of orthogonal coordinates is used. One is the distance from the surface, and the other two are measured along the lines of curvature of the surface. The definition of this system is not quite unique, but becomes so for vanishingly thin boundary layers. This is demonstrated for a conical surface. The boundary-layer equations set up are very similar to the normal boundary-layer equations except for two additional terms which depend upon the principal curvature of the surface coordinate lines. The necessary and sufficient condition that the curvature terms disappear is that the boundary layer should be along the surface of a cylinder or a plane. The form assumed by the equations when the surface is a solid of revolution is derived. A discussion of the flow in the vicinity of a stagnation point on a general surface will be given in part II.

Alb. Betz, Germany

3641. Oudart, A., Calculation of the laminar or turbulent boundary layer in a compressible fluid. Modern semi-empirical methods and the papers by Dr. Ing. Alfred Walz (in French), *Publ. sci. Min. Air, France* no. 233, vii + 101 pp., 1949.

Paper gives an extensive survey of the various integral methods for the calculation of laminar and turbulent boundary layers in an incompressible as well as a compressible fluid. In all the methods developed for the compressible laminar boundary layer, the Prandtl number of the flow is taken to be unity and the boundary is insulated. This gives the usual simplification of eliminating the temperature distribution algebraically in terms of the velocity distribution. Various methods of approximating the velocity profiles are described, using principally a one-parameter family of curves. The two-parameter method is mentioned only briefly. Discussions in the turbulent case include several semi-empirical approaches. C. C. Lin, USA

3642. Lock, R. C., The velocity distribution in the laminar boundary layer between parallel streams, *Quart. J. Mech. appl. Math.* **4**, part 1, 42–63, Mar. 1951.

A method is given for obtaining the velocity distribution of steady flow in the free laminar boundary layer separating two parallel streams of incompressible viscous fluids which have different densities and viscosities. The solutions depend on the ratio of the velocities of the two streams and on the ratio of the products of the density and viscosity of the respective streams, $\rho\mu = \rho_2\mu_2/\rho_1\mu_1$. Numerical results are given for the case where the lower fluid is at rest for four values of $\rho\mu$ (one of which corresponds to flow of air over water at 10 C). Calculations are also made for $\rho\mu = 1$ and a velocity ratio of 0.5. Use of the momentum equation is also made to obtain an approximate solution to this problem. Simon Ostrach, USA

3643. Deissler, R. G., Investigation of turbulent flow and heat transfer in smooth tubes, including the effects of variable fluid properties, *Trans. Amer. Soc. mech. Engrs.* **73**, 2, 101–107, Feb. 1951.

For the case described by title, a single expression is derived for the velocity distribution in the wall region (laminar sublayer) through the assumption that the eddy diffusivity $\epsilon = n^2uy$ where n^2 is an experimentally determined constant and u is the local velocity at a distance y from the wall. The resulting velocity distribution, which is not appreciably different from that suggested by von Kármán [title source, **61**, 705–710, 1939], was verified experimentally for isothermal flow. The velocity-distribution relations are extended to the flow of a gas (Prandtl no. = 1.0) with heat transfer and variable fluid properties. No comparison is made with experimental data for this case. Assuming constant shearing stress and constant heat flux (per unit area) between the wall and tube axis, the effects of fluid-property variation on heat transfer and fluid friction are found. It is found that these effects can be absorbed into the relations for "isothermal" conditions by the evaluation of the fluid properties at approximately the "film" temperature, i.e., the average of wall and bulk temperatures. The prediction for friction in isothermal flow agrees well with one set of experimental data. Approximate agreement is also obtained for a limited amount of heating data for air flow through tubes. Carl Gazley, Jr., USA

3644. Corrsin, S., On the spectrum of isotropic temperature fluctuations in an isotropic turbulence, *J. appl. Phys.* **22**, 4, 469–473, Apr. 1951.

Paper develops the form of equilibrium frequency spectrum for homogeneous isotropic scalar temperature fluctuations in a field of homogeneous isotropic turbulence in an incompressible fluid. Density changes due to pressure and temperature fluctuations are assumed negligible, as is heat generated by viscous dissipation.

Incompressible energy equation yields a differential equation relating the scalar double correlation between temperature fluctuations and the characteristic scalar of the triple vector correlation between two temperatures and a velocity. By taking three-dimensional Fourier transforms of these correlation functions, this is converted to an equation for the temperature spectral function. The form of the latter is deduced for the low wave-number range where a power-series expansion in k indicates a k^2 start of the spectrum for $k \to 0$; a result consistent with the proved invariance of the second moment of the double temperature correlation. Application of dimensional reasoning and the concept of an eddy conductivity expressed in terms of the spectral function following Heisenberg, yields for the latter a $k^{-5/3}$ and k^{-7} variation, respectively, in the intermediate and high wave-number ranges. Some discussion of relative cut-off wave numbers for the temperature and velocity spectra is included.

Lawrence M. Grossman, USA

3645. Szablewski, W., Calculation of turbulent pipe flow on the basis of the mixing length hypothesis (in Germany), *Z. angew. Math. Mech.* **31**, 4/5, 131–142, Apr./May 1951.

The differential equation for fully developed turbulent flow in a pipe is integrated using Prandtl's mixing length relations. The integration starts at the wall and considers two regions, the laminar sublayer and a region where the order of magnitude of laminar and turbulent friction is the same. The solution for these two regimes is used as approximation for the remainder of the pipe flow. Appropriate values of the empirical coefficients give good agreement with experimental results of Nikuradse.

H. P. Liepman, USA

3646. Bass, J., Space and time correlations in homogeneous turbulence (in French), *C. R. Acad. Sci. Paris* **232**, 21, 1905–1906, May 1951.

The turbulent motion of an incompressible fluid is described,

using double and triple velocity-correlation functions that involve temporal as well as spatial displacements. By assuming the turbulence to be homogeneous in space and stationary in time, two equations relating these space-time correlations are obtained, analogous to the Kármán-Howarth equation for space correlations. The somewhat simpler spectral forms of these equations are also given. Author considers that, from the form of the correlation functions, it is necessary that initial variation of correlation with time is due in effect to convection of turbulence by the mean stream, i.e., that in a coordinate system moving with the mean stream, the correlations are independent of time displacement. This prediction is confirmed by experiment, but reviewer does not believe that the explanation given here is the correct one.

A. A. Townsend, England

3647. Van Driest, E. R., Turbulent boundary layer in compressible fluids, *J. aero. Sci.* **18,** 3, 145–160, 216, Mar. 1951.

"The continuity, momentum, and energy differential equations for turbulent flow of a compressible fluid are derived, and the apparent turbulent stresses and dissipation function are identified. A general formula for skin friction, including heat transfer to a flat plate, is developed for a thin turbulent boundary layer in compressible fluids with zero pressure gradient. Curves are presented giving skin-friction coefficients and heat-transfer coefficients for air for various wall-to-free-stream temperature ratios and free-stream Mach numbers.

"In the special case when the boundary layer is insulated, this general formula yields skin-friction coefficients higher than those given by the von Kármán wall-property compressible-fluid formula but lower than those given by the von Kármán incompressible-fluid formula. Heat transfer from the boundary layer to the plate generally increases the friction and heat-transfer coefficients." (Author's summary)

The detailed mathematical development uses such simplifying assumptions as a very thin boundary layer with fully developed turbulence from the plate leading edge at all Mach numbers from two to ten, no external pressure gradient, and a Prandtl number of unity. In view of these, the conclusions are mainly of academic interest until validated by extensive experimental data. The only comparison that is given between the present theory and results is for mean skin-friction coefficients. The measured results show not only a wide quantitative spread but also a considerable difference in trend.

Ione D. V. Faro, USA

Aerodynamics of Flight; Wind Forces

(See also Revs. 3442, 3475, 3526, 3527, 3619, 3679)

3648. Shinbrot, M., A least squares curve fitting method with applications to the calculation of stability coefficients from transient response data, *Nat. adv. Comm. Aero. tech. Note* 2341, 52 pp., Apr. 1951.

Assume the forcing function and transient response are available, in graphical form or tabulated at intervals in time, for a system described by linear differential equation $P(D)q(t) = Q(D)F(t)$, where P and Q are polynomials in differential operator D, q is the response, and F the forcing function. Author presents method of obtaining coefficients of P and Q such that q best fits given number of points from known response. Given approximate fitted response, author uses Taylor's series to obtain set of linear equations which give improved fit when solved by least squares. Process is iterative.

Method has fundamental theoretical advantage over others presented since correct quantity, difference between calculated and known responses is minimized. Applications are given to obtaining stability coefficients of two-degree-of-freedom longitudinal airplane motion from pitching response to various types of forcing function. Author's assertion that examples may be understood without reading development of method seems doubtful to this reviewer, especially in view of unfortunate typographical errors.

Albert A. Schy, USA

3649. Johnson, H. S., and Hagerman, J. R., Wind-tunnel investigation at low speed of lateral control characteristics of an untapered 45° sweptback semispan wing of aspect ratio 1.59 equipped with various 25-per cent-chord plain ailerons, *Nat. adv. Comm. Aero. tech. Note* 2316, 30 pp., Mar. 1951.

Effects of aileron span and spanwise location of the ailerons on the low-speed lateral control characteristics of a sweptback wing equipped with a 0.25-chord plain aileron are presented. The rolling effectiveness of the ailerons increased with increasing aileron span and an aileron of a given span was most effective when located outboard on the wing. At high angles of attack, large adverse yawing moments were measured. The ratio of adverse yawing moment to rolling moment was minimized by locating the aileron outboard on the wing. The hinge-moment slope parameters were little affected by changes in aileron span or spanwise location. The variation of rolling moment and hinge moment with aileron deflection could be predicted with good accuracy by existing empirical relationships [AMR **1,** Rev. 1678; **2,** Rev. 1309].

Robert M. Crane, USA

3650 Fischel, J., and Hagerman, J. R., Effect of aspect ratio and sweepback on the low-speed lateral control characteristics of untapered low-aspect-ratio wings equipped with retractable ailerons, *Nat. adv. Comm. Aero. tech. Note* 2347, 36 pp., May 1951.

Investigations have been made of lateral control characteristics of four wings equipped with retractable ailerons (spoilers). These tests were conducted in the LMAL 300-mph 7 × 10 tunnel. Models included three straight, untapered wings of aspect ratios 4.13, 2.13, and 1.13, and an untapered wing of aspect ratio 1.09 and 45° sweepback. Tests were conducted because previous experimental investigations have shown substantial advantages for retractable ailerons in high-speed aircraft. Among these are favorable yawing moments, preservation of control or high subsonic Mach numbers, and ability to use full span flaps. Tests showed low effectiveness and large range of reversed effectiveness for the lowest aspect ratio, with improved characteristics as aspect ratio increased. Tests on sweptback wing showed effectiveness that compared well with plain flap ailerons, particularly for the case where retractable ailerons were located on inboard portion of wing.

Wilbur L. Mitchell, USA

3651. Laitone, E. V., Rigid-body dynamics of aircraft including nonstationary aerodynamics, *Douglas Aircr. Co. Rep.* SM-13820, 58 pp., Jan. 1951.

Report is concerned primarily with symmetrical longitudinal response of rigid aircraft to rapid elevator deflections, although antisymmetric motions are formulated. Perturbation equations of motion for rigid aircraft are rederived and are simplified to a form for which practical solutions may be carried out. Conditions for validity of the simplifications are discussed, and it is shown that variations in forward speed may be neglected in calculating response of conventional airplanes and missiles to sufficiently rapid control-surface movements. Differential equations for aircraft velocities are written with conventional stability derivatives as coefficients. Five methods are discussed for determining response when arbitrary control motions are considered. Included is a newly developed graphical method permitting rapid solutions

when response to unit step function is known [AMR **4**, Rev. 2782].

Evaluation of stability derivatives, or their equivalents, from dynamic flight tests is outlined, and a critical survey of previous methods [for instance, AMR **1**, Rev. 154] is given. The effect of nonstationary aerodynamics on conventional stability derivatives is examined [*J. aero. Sci.* **18**, 3, 214–216, Mar. 1951], and errors in transformation from flutter to stability axes by previous investigators are corrected. Method for approximate finite span correction of apparent mass terms, first proposed by Jones [*J. aero. Sci.* 9, 1, 8–10, Nov. 1941] is reproposed. In transonic range, flight tests are considered to be the only practical method for determining frequency-response spectrum from which, in turn, response to arbitrarily prescribed control-surface motions may be determined. Author does not explicitly assume linearity in transonic realm, and reviewer believes that nonlinearity of unsteady compressible-flow equations, even with velocities as independent variables, suggests invalidity of superposition required by Fourier techniques, as proposed by author, for this range.

T. F. O'Brien, USA

3652. Driggs, I. H., Aircraft design analysis, *J. roy. aero. Soc.* **54**, 470, 65–116, Feb. 1950.

A comprehensive description of the methods devised by author to enable rapid estimation of the performance of a series of airplanes. The particular needs of this type of analysis were met by the construction of a kind of two-dimensional slide rule, employing logarithmic graphs together with suitably engraved transparent templates and overlays. The computations, which may involve corrections of the weight, altitude, thrust, etc., are performed by manipulations of these graphs.

In addition to the method of computation, paper presents a considerable amount of design information in the form of 55 graphs, and includes examples showing the variation of performance with design parameters in typical cases.

Robert T. Jones, USA

3653. Pindzola, M., Supersonic tests of conventional control surfaces on a double-wedge airfoil, *J. aero. Sci.* **17**, 4, 204–210, Apr. 1950.

Describes results of supersonic tunnel tests on double-wedge airfoil of chord 6 in., 7% thick, fitted with flaps of length 0.15 c and 0.30 c, deflected over range $\pm 0.20°$. A range of attack angle was investigated. Pressure measurements and schlieren photographs were made at M = 1.38, 1.48, and 1.58. Effect of sting on pressure measurements was experimentally eliminated.

Experimental results compared with Busemann's second-order linear theory and the shock-expansion theory (cf. *Rep. Mem.* 2004, 1943, and 2101, 1944) are in reasonably good agreement except where nose shock became detached. At $\alpha = -7°$, this caused a high pressure at leading edge which decreased steadily to theoretical value, resulting in increased $dC_L/d\alpha$ and C_D and forward shift of center of pressure. During nose-shock detachment, $dC_L/d\alpha$ increased rapidly but only increased slowly when shock became fully detached. Where $C_L = 0$, experimental curve of $C_L - \alpha$ was nearly linear. Boundary-layer separation reduced lift. No-lift angle was best predicted by second-order linear theory.

$C_D - \alpha$ curve was parabolic where lift curve was linear. Agreement with second-order linear theory was good in this region, but deviated where unpredicted flow phenomena occurred. Separation increased, but detached nose shock decreased C_D. Curves given, however, represent pressure drag only and do not include skin-friction drag. Center of pressure was at infinity at $\alpha_0°$ but approached 0.5 c as α increased. $dC_M/d\alpha$ was increased by flow pattern irregularities and agreed best with second-order linear theory.

C_H (about hinge point) approximated accurately under normal flow conditions to second-order linear theory. Separation reduced C_H. It is thus seen that partial separation had little effect except on C_H, but complete separation appreciably affected all characteristics, as above. Effect of flap deflection δ at constant incidence gave positive value of $dC_L/d\delta$ but was experimentally smaller than predicted theoretically.

Only representative results of complete tests are given.

John G. Ross, England

3654. Harrington, R. D., Full-scale-tunnel investigation of the static-thrust performance of a coaxial helicopter rotor, *Nat. adv. Comm. Aero. tech. Note* 2318, 23 pp., Mar. 1951.

As part of a research program on different helicopter rotor configurations, the NACA presents experimental static thrust data from two coaxial rotors and a comparison with theory. Tests include operation of single rotors (halves of the coaxial) and a study of the effect of differential cyclic control on hovering thrust-power ratio.

Hovering performance of the coaxial rotor is found no different from that of a single rotor of same solidity. The coaxial rotors require considerably more power for a given lift than the single rotors with far removed axes.

W. R. Woodward, USA

3655. de Guillenschmidt, P., Calculation of the bending stresses in helicopter rotor blades, *Nat. adv. Comm. Aero. tech. Memo.* 1312, 32 pp., Mar. 1951.

First step is to determine the normal functions which represent the natural modes of vibration of the rotating blade. Making use of the orthogonality properties of these functions, a solution of the governing differential equation is obtained in the form of a series of the normal functions. Author claims that sufficient accuracy is obtained by using only first three normal functions. Numerical examples are given for two different helicopter rotor blades.

Dana Young, USA

3656. Greenberg, H., A survey of methods for determining stability parameters of an airplane from dynamic flight measurements, *Nat. adv. Comm. Aero. tech. Note* 2340, 49 pp., Apr. 1951.

Reduction of dynamic flight measurements to transfer coefficients, which usually incorporate a number of stability derivatives, is becoming an increasingly necessary type of analysis due to need of "true-to-scale" measurements in calculating motions of aircraft. Methods surveyed in this report have all been employed in practice but as yet no definite judgment is possible regarding their accuracy or relative utility. Author suggests the computation of the "error ratios" of the parameters involved as future work that might alleviate this situation. Some discussion of the problem of obtaining stability derivatives from the transfer coefficients is given and a discussion of the effects of aerodynamic lag is included.

Arthur L. Jones, USA

3657. Babister, A. W., Measurement of the pressure distribution on swept back wings with trailing edge split flaps, *Coll. Aero. Cranfield Rep.* **43**, 12 pp., Mar. 1951.

Span and chord pressure-distribution data are presented for two 45° sweptback untapered wings of aspect ratio 4 and 2. Results are given for no flaps, 45° full-span flaps, 45° inboard-semispan flaps, 45° outboard-semispan flaps for the aspect-ratio-4 wing at 0.5×10^6 Reynolds number, and no flaps, 30° full-span flaps, 30° inboard-semispan flaps for the aspect-ratio-2 wing at 0.6×10^6 Reynolds number. Angle of attack range of tests range from 0 to 30°. Flaps are found less effective on swept than straight

wings, stalling incidence is lower, and stall more sudden than on swept wing without flaps. Limited comparison is given of measured span loading at low incidence with lifting-line theory calculations. W. S. Aiken, Jr., USA

3658. **Young, A. D., The induced drag of flapped elliptic wings with cut-out and with flaps that extend the local chord,** *Aero. Res. Counc. Lond. Rep. Mem.* 2544, 12 pp., Feb. 1942, published 1951.

Calculations have been made of the induced drag of flapped elliptic wings covering a range of aspect ratios from about 4 to about 12, a range of flap spans from 0.2 to 1.0 of the wing span, and a range of flap cutouts from 0 to 0.6 of the wing span. Results are presented in charts in a form convenient for application.

Calculations have also been made of the induced drag of elliptic wings of an aspect ratio of about 6 with flaps that extend the local chord by 40% when in operation; flap spans of 0.26, 0.50, and 0.77 of the wing span were examined.

It is concluded that for a given net flap span and lift increment, minimum induced drag will be obtained with a cutout of about 0.1 wing span. The effect of local chord extension due to a flap was found to be negligible. These results apply strictly to elliptic wings but they probably apply with fair accuracy to wings of taper ratio of the order of 2:1. From author's summary

3659. **Jackson, S. B., Free-flight tests on kites in the 24-ft wind tunnel,** *Aero. Res. Counc. Lond. Rep. Mem.* 2599, 18 pp., Mar. 1942, published 1951.

This appears to be a continuation at higher wind-tunnel speeds of previous tests not referenced. A ratio of net lift to drag is defined as the kite lifting efficiency and is computed for all test conditions. Several basic and modified kites were tested over the whole stable range and up to the highest safe wind speed. Various combinations of bridles and stiffening methods were employed, yielding a maximum lifting efficiency of 2.5 and C_L of 0.9 based on the surface area. Since there are not many careful investigations of kites, this article should be of interest to those trying to use kites for specific purposes. Robert S. Ross, USA

3660. **Whitaker, W. E., Jr., and Bryce, P. W., Jr., Effect of an increase in angle of dead rise on the hydrodynamic characteristics of a high-length-beam-ratio hull,** *Nat. adv. Comm. Aero. tech. Note* 2297, 42 pp., Feb. 1951.

An increase in angle of dead rise from 20° to 40° increases the range of stable trim between the upper and lower trim limits of stability, increases the range of center-of-gravity position available for satisfactory take-off stability, and substantially improves the spray characteristics. The water resistance is increased appreciably in the planing range so that the take-off time and distance are increased approximately 25% and 30%, respectively. The over-all rough-water landing behavior is improved. The maximum vertical and angular accelerations are reduced approximately 55% and 30%, respectively. From authors' summary

3661. **McArver, A. Ethelda, Water-landing investigation of a model having heavy beam loadings and 0° angle of dead rise,** *Nat. adv. Comm. Aero. tech. Note* 2330, 19 pp., Apr. 1951.

A model having heavy beam loadings and an angle of dead rise of 0° was subjected to smooth-water impacts. The tests were made at fixed trims of 6°, 9°, 12°, 15°, 30°, and 45° for a range of flight-path angles from approximately 2° to 22°. Most of the tests were made at a beam-loading coefficient of 18.8, but a few impacts were made at a beam-loading coefficient of 36.5. The data from the higher beam loading were too few, however, to establish quantitatively any effects of variation of beam loading; consequently, trends have been noted only from the data obtained with a beam-loading coefficient of 18.8.

Time histories of horizontal and vertical displacements, vertical velocity, vertical acceleration, and pitching moment were obtained. Results are presented as plots showing the variation of the experimentally determined quantities converted to nondimensional coefficients with flight-path angle at water contact. The impact-lift coefficient increases with increasing flight-path angle at water contact, but shows no marked variation with trim angle. The draft coefficients at maximum vertical acceleration and at maximum immersion increase linearly with flight-path angle at water contact and, at maximum vertical acceleration, increase with increasing trim; however, at maximum immersion, the draft coefficient is not appreciably affected by trim.

From author's summary

3662. **Dods, J. B., Jr., Estimation of low-speed lift and hinge-moment parameters for full-span trailing-edge flaps on lifting surfaces with and without sweepback,** *Nat. adv. Comm. Aero. tech. Note* 2288, 47 pp., Feb. 1951.

Work compares results of lifting-line and modified lifting-surface theory. Proposed theoretical method for preliminary design estimates determines lift by Weissinger lifting-line theory [AMR **3**, Rev. 917] and determines induced camber, and chordwise load distribution by Falkner lifting-surface theory. Design charts are given for induced camber, and sample calculations are included.

Method usually gives better accuracy than that in *NACA T.N.* 1506 [AMR **1**, Rev. 699] especially for C_{h_β}. Method is limited to constant per cent chord full span flaps, with internally sealed nose. A. W. Babister, England

3663. **Williams, J., Some improvements in the design of thick suction aerofoils,** *Aero. Res. Counc. Lond. curr. Pap.* 31, 12 pp., Mar. 1950.

Airfoil sections designed to have a large pressure rise at one point on the surface, at which suction is applied, have exhibited unsatisfactory flow characteristics when tested. The source of these difficulties was apparently extreme surface curvature in the vicinity of the suction slot. It is shown that this curvature can be reduced by extending the design theory to include suction slots of finite width. Gerald E. Nitzberg, USA

3664. **Naeseth, R. L., and O'Hare, W. M., Effect of aspect ratio on the low-speed lateral control characteristics of unswept untapered low-aspect-ratio wings,** *Nat. adv. Comm. Aero. tech. Note* 2348, 47 pp., May 1951.

A low-speed investigation was made to determine the lateral control characteristics for a series of unswept, untapered, complete wings of aspect ratios 1.13, 2.13, 4.13, and 6.13. The wings were equipped with 0.25-chord sealed ailerons of various spans and of various spanwise locations.

The variation of experimental aileron effectiveness with wing aspect ratio was not predicted accurately for all spans of aileron by any one of the three theoretical methods with which a comparison was made. Design charts based on the experimental results are presented for estimating the aileron effectiveness for low-aspect-ratio, untapered, unswept wings.

From authors' summary

3665. **Weissinger, J., On integrodifferential equations of the type of Prandtl's wing equation** (in German), *Math Nachr.* **3**, 316–326, 1950.

Consider the integral equation

$$G(\theta) = L(\theta)\left\{\alpha(\theta) - (2\pi)^{-1}\int_0^{\pi} G'(\psi)[\cos\psi - \cos\theta]^{-1}d\psi\right\} \quad [*]$$

Multhopp's procedure [*Luftfahrtforschung* **15**, 153–169, 1938] for finding an approximate solution is to approximate $G(\theta)$ by a trigonometric polynomial $\gamma(\theta)$ passing through the points $(\theta\nu = \nu\pi/(m+1), \gamma_\nu)$, $\nu = 1, \ldots, m = 2^k - 1$. Equation [*], for $\theta = \theta_\nu$, then reduces to a set of linear equations in the m unknowns $\gamma_1, \ldots, \gamma_m$. The purpose of this paper is to show that as $m \to \infty$, the values $\gamma_\nu \to G(\theta_\nu)$. This is shown under the hypothesis that $\sin\theta/L(\theta)$ and $\alpha(\theta)$ are continuous and piecewise continuously differentiable and that $L(\theta)/\sin\theta \geq c > 0$ for $0 \leq \theta \leq \pi$. The author next considers a generalization of the integral equation [*] and proves, under restrictions easily satisfied in applications to wing theory, that a solution exists and that the Multhopp procedure converges. J. V. Wehausen, USA

3666. Malvestuto, F. S., Jr., and Hoover, D. M., Supersonic lift and pitching moment of thin sweptback tapered wings produced by constant vertical acceleration. Subsonic leading edges and supersonic trailing edges, *Nat. adv. Comm. Aero. tech. Note* 2315, 44 pp., Mar. 1951.

Theoretical approximations to the nondimensional lift and pitching moment produced by constant vertical acceleration are derived for a series of thin sweptback tapered wings with streamwise tips. Analysis is essentially the application of a recently published solution of the linearized time-dependent wave equation for wings in accelerated motion [AMR **4**, Rev. 2124]. Results are independent of camber and thickness and are applicable for a range of supersonic speed for which the wing is wholly contained between the Mach cones springing from the wing apex and from the trailing edge of the root section (subsonic leading edge and supersonic trailing edge).

Design curves are presented which permit rapid estimation of the derivatives for given values of aspect ratio, taper ratio, Mach number, and leading-edge sweep.

From authors' summary by T. F. O'Brien, USA

3667. Malvestuto, F. S., Jr., and Margolis, K., Theoretical stability derivatives of thin sweptback wings tapered to a point with sweptback or sweptforward trailing edges for a limited range of supersonic speeds, *Nat. adv. Comm. Aero. Rep.* 971, 14 pp., 1950.

See AMR **2**, Rev. 225.

3668. Greidanus, J. H., and van Heemert, A., Chordwise downwash distribution of an infinite wing of constant chord with a periodic spanwise distribution of vorticity, in oblique flow, *Nat. LuchtLab. Amsterdam Rap.* F. 78, 24 pp., Nov. 1950.

Simplified subsonic lifting-surface theory, known as Prandtl's lifting-line theory, breaks down when applied to cases of oblique flow. Paper reinspects the simplifications leading to Prandtl's result and presents a modified lifting-line theory that will allow the calculation of induced cambers in oblique flow along sections where a spanwise variation of vorticity occurs. Resulting expressions are checked against rigorous lifting-surface procedure by applying both to a sweptback two-dimensional wing having a periodic spanwise vorticity distribution. The agreement is good provided the frequency is sufficiently low.

Harvard Lomax, USA

3669. Järvineva, E., The steepest angle of climb of jet aeroplanes in unaccelerated flight, *Aircr. Engng.* **23**, 265, 76–77, Mar. 1951.

Equilibrium of forces in lift and drag direction for an airplane in steady climb is set up and the equations simplified to relate the thrust-to-weight ratio, angle of climb, maximum lift-drag ratio, and ratio of climbing velocity to velocity in level flight corresponding to maximum lift-drag ratio. Resulting equations can be represented graphically in a polar vector diagram, which relates the same quantities. The analysis is made under the assumption that the lift-drag polar is parabolic, that the powerplant thrust is independent of flight velocity, that the airplane acceleration in the flight direction can be neglected, and that the drag coefficient is not affected by Mach number, which essentially limits the analysis to velocities less than the velocity corresponding to the critical Mach number of the airplane. The polar diagram is very convenient to use and will permit calculation of the climbing speed and rate of climb, knowing the level flight speed corresponding to maximum lift-to-drag ratio.

H. P. Liepman, USA

3670. Skoog, R. B., and Brown, H. H., A method for the determination of the spanwise load distribution of a flexible swept wing at subsonic speeds, *Nat. adv. Comm. Aero. tech. Note* 2222, 46 pp., Mar. 1951.

Using a relaxation method, the interaction between aerodynamic loading and elastic deformation is expressed in a power series which can be approximated by a geometrical series. The solution is expressed in a convenient form such that the amount of detailed computing involved when extensive aeroelastic calculations for many flight conditions are desired is reduced to that for a single set of flight conditions.

Sample computing forms and a numerical example are presented to illustrate the method.

From author's summary by H. G. Loos, Holland

Aeroelasticity (Flutter, Divergence, etc.)

(See Revs. 3593, 3617, 3626, 3628, 3629, 3670)

Propellers, Fans, Turbines, Pumps, etc.

(See also Revs. 3474, 3614, 3675)

3671. Panetti, M., A limit hypothesis for the calculation of the characteristic of axial compressors (in Italian), *Aerotecnica* **31**, 1, Spec. no., 5–10, Feb. 1951.

See AMR **4**, Rev. 3358.

3672. Squire, H. B., and Winter, K. G., The secondary flow in a cascade of airfoils in a nonuniform stream, *J. aero. Sci.* 18, 4, 271–277, Apr. 1951.

If the flow upstream of a cascade of airfoils is parallel but not uniform in the spanwise direction, the flow downstream is, in general, no longer a parallel flow. The flow across the main flow direction is the so-called secondary flow. In order to explain and to compute this secondary flow in a first-order approximation, there is, as the authors indicate, no need to take viscosity into account. Comparison between calculated results and tests show that the theory may lead to useful results. The secondary flow is the reason for losses which can be compared to the induced drag of a single airfoil of finite span, in addition to the pressure losses which are the results of the profile drag and are of comparable magnitude.

The theoretical derivations are based on small deviations from potential flow over a large range and on small deflection. The comparison is made with a 90° deflection by a cascade in the corner of a wind tunnel. This may explain some of the differences found. Friedrich Weinig, USA

3673. Laidlaw, W. R., Ejector theory and its application to induction type wind tunnels *Nat. Res. Counc. Canad. mech. Engng. Rep.* MA-232, 31 pp., Aug. 1950.

A theory applicable both to tunnels driven by a high pressure reservoir and to tunnels using the exhaust jet of a gas turbine is developed. The design of the gas-turbine-type tunnel is treated in detail. Included is a discussion of the static pressure at the exit of the high-velocity driving jet. (Reviewer's note: equation (19) seems to contain misprints.) Results of experiments on a scale model of a gas-turbine-type tunnel, driven by compressed air, are presented and compared with theoretical predictions.

Neal Tetervin, USA

3674. Katto, Y., On the ejector used for locomotives (in Japanese), *Rep. Inst. Sci. Technol. Tokyo* **4**, 44–49, Jan.-Feb. 1950.

An ejector with a large mixing chamber, similar to the type used for a locomotive to exhaust its combustion gases, is treated theoretically on conditions that the first and the second fluids are the same and incompressible. "Inflow coefficient" is used as usual, but the author analyzes the coefficient and expresses it as a function of mixing ratio and dimensions of apparatus. Tollmien's theory of turbulent mixing region formed by a jet is used to solve the problem.

Norimune Soda, Japan

3675. Leist, K., and Knörnschild, E., Exhaust turbine and jet propulsion systems, *Nat. adv. Comm. Aero. tech. Memo.* 1294, 48 pp., Apr. 1951.

Translation of *ZWB Forschungsber.* 1069, May 1939.

3676. Trout, A. M., Theoretical turbojet thrust augmentation by evaporation of water during compression as determined by use of a Mollier diagram, *Nat. adv. Comm. Aero. tech. Note* 2104, 41 pp., June 1950.

3677. Englert, G. W., and Ross, A. O., Investigation of first stage of two-stage turbine designed for free-vortex flow, *Nat. adv. Comm. Aero. tech. Note* 2107, 30 pp., June 1950.

3678. Weinig, F., Flight performance of a jet power plant. III. Operating characteristics of a jet power plant as a function of altitude, *Nat. adv. Comm. Aero. tech. Memo.* 1258, 40 pp., Translation from German, May 1951.

Performance of a jet engine is analyzed on the assumption that the pressure coefficient of compressor (and turbine) varies linearly with the flow coefficient. Output and fuel consumption at off-design are compared with design values.

Hsuan Yeh, USA

3679. Cleaver, A. V., Rockets and assisted take-off, *J. roy. aero. Soc.* **55**, 482, 87–102, Feb. 1951.

Paper discusses the fundamentals of rocket propulsion and some rocket engineering problems such as propellant choice, feed systems, ignition, and cooling. Some of the history of rocket-assisted take-off is cited.

The de Havilland Sprite, a hydrogen-peroxide-fueled assisted take-off unit that provides 56,000 lb/sec total impulse and a maximum thrust of about 5,000 lb from a full weight of 915 lb and empty weight of 350 lb, is described. Only very general comparisons with other forms of take-off assistance are made, although it is noted that the rocket can provide "almost unlimited" thrust, thus providing advantages that can be obtained in no other way. Except for the description of the Sprite, there is little new rocket information in the paper.

W. T. Olson, USA

Flow and Flight Test Techniques

(See also Revs. 3535, 3596, 3649, 3653, 3654, 3662, 3673, 3710)

3680. Drees, J. M., and Hendal, W. P., Airflow patterns in the neighbourhood of helicopter rotors, *Aircr. Engng.* **23**, 266, 107–111, Apr. 1951.

Paper describes results obtained from wind-tunnel smoke tests with a two-blade model rotor of about 1-ft diam. By changing the pitch-setting, tip-speed ratio, and incidence of the rotor axis, all possible flight conditions could be simulated.

It is shown that the three different working states of vertical flight, namely, the propeller-working state, the vortex-ring state, and the windmill-brake state, also exist in forward flight. The uncontrollable behavior of the helicopter in partial power-off descent at low forward speeds is explained by the existence of an "airbody" around the rotor due to a spread of the slipstream. Paper is illustrated by several excellent smoke photographs. It appears that the Figure numbers 10 and 11 have been reversed.

G. J. Sissingh, England

3681. Castagna, A., Comparison between the characteristics of continuous and intermittent supersonic wind tunnels (in Italian), *Termotecnica* **5**, 1, 41–44, Jan. 1951.

A comparison of the theoretical performance of different types of continuous and intermittent supersonic wind tunnels is made. Expressions for the adiabatic work done are given in terms of the pressure ratio required for driving the continuous type of return flow tunnel. The useful working time and the time to complete a cycle for the intermittent type of tunnel are given in terms of the ratio of the power available to the power required to drive the continuous type of return flow tunnel.

Little derivation of the expressions quoted is given, and the reviewer remarks that formulas 1, 2, and 4 are clearly in error.

G. M. Lilley, England

3682. Cowdrey, C. F., A note on the calibration and use of a shielded hot-wire anemometer for very low speeds, *J. sci. Instrum.* **27**, 12, 327–329, Dec. 1950.

Calibration of air-speed measuring instruments at low speeds, of the order of 1 fps, is very troublesome. Consequently, this article will interest those concerned with the measurement of low speeds, particularly those employing a heat-loss type of anemometer such as the one used here. A special whirling-arm technique is described whereby the difficulty from convection currents is largely overcome and the uncertainty in the swirl correction is reduced. Curves showing the percentage swirl are given. The anemometer is the heated-thermocouple type with heater and thermocouple in a fine silica tube. Details on the construction are lacking. A curve shows the performance up to about 6 fps, and 4 graphs show the effects of pitch and yaw.

G. B. Schubauer, USA

3683. Giese, J. H., Bennett, F. D., and Bergdolt, V. E., A simple interferometric test for conical flow, *J. appl. Phys.* **21**, 12, 1226–1231, Dec. 1950.

Flows in which all flow quantities remain constant on the rays of a pencil (conical flows) furnish interferograms whose fringe shifts δ satisfy a functional constraint. In a Cartesian coordinate system whose origin is at the vertex and whose x-axis in the direction of the transluminating beam, $\delta(y, z)/z$ is a function of y/z alone. For any Taylor-Maccoll flow this function can be calculated a priori [Kopal, Z., "Tables of supersonic flow around Cones," Cambridge, Mass., 1947]. $\delta(y, z)/z$ vs. y/z plots of interferograms, obtained from range firings of cal. 0.30 cone-cylinder models at atmospheric conditions, are analyzed in the light of

these two facts. Plotted points fall into satisfactorily narrow bands following theoretical Taylor-Maccoll curve. Small systematic deviations both from conicalness and from the Taylor-Maccoll model are noted but not examined as to their significance. T. J. Weyl, USA

3684. Bennett, F. D., Optimum source size for the Mach-Zehnder interferometer, *J. appl. Phys.* 22, 2, 184–190, Feb. 1951.

The formation of undisturbed fringes by an ideal interferometer with an extended source is treated analytically. The fringe-clarity condition is expressed as an inequality, which is analyzed. The optimum source area is determined and two numerical examples are considered. N. H. Johannesen, England

3685. Black, J., and Mediratta, O. P., Supersonic flow investigations with a "hydraulic analogy" water channel, *Aero. Quart.* 2, part 4, 227–253, Feb. 1951.

Authors give a quite comprehensive survey on the analogy between flow of shallow water with a free surface and two-dimensional gas flow. Their aim is to determine the limitations of the analogy. To this end they compare the hydraulic theory with their own experiments in a water channel, which are described and illustrated by numerous good photographs of the flow patterns of typical cases. It can be shown that qualitative analogy is very good. Quantitative analogy can be reached by comparing the hydraulic values with the "hydraulic gas," a fictive gas with the ratio of the specific heats of 2. This analogy is only valid, however, as long as no "hydraulic jumps" are involved. Those jumps which find their qualitative analogy in shock waves of gases are governed by a law which has no quantitative analogy to shock waves, even with the hydraulic gas. Authors point out convenience of using water channel to demonstrate flow patterns of gas flow. They stress, however, the lack of quantitative agreement.

Paper is significant in showing that it is doubtful if attempts to find solutions of problems in the field of gas dynamics by hydraulic tests are worth while. Heinrich J. Ramm, USA

3686. Hilton, W. F., Wind-tunnel tests for temperature recovery factors at supersonic velocities, *J. aero. Sci.* 18, 2, 97–100, Feb. 1951.

Author presents adiabatic surface temperatures measured on an ogival body of revolution at Mach numbers of 1.73 and 2.00. Data are presented for angles of attack from zero to four degrees. Free-stream Reynolds numbers for the Mach number 2 run varied from 9.3×10^6 to 9.9×10^7. No information is given as to the Reynolds number range of the Mach 1.73 run.

The data obtained at zero angle of attack are presented in terms of temperature-recovery factor and agree well with results obtained on flat plates [see AMR 4, Rev. 1703]. Unfortunately, the data obtained at angles of attack greater than zero were not reduced to recovery-factor form and cannot be compared easily with the results of other investigators. Glen Goodwin, USA

3687. Beede, H. M., and Droms, C. R., A simplified thermocouple for temperature measurements in high velocity gas streams, *Instruments* 24, 3, 338–341, Mar. 1951.

A thermocouple with a high recovery factor has been developed. The test facilities and characteristics of several variations of the design are described. The best designs have a recovery factor of 0.95 ± 0.01 over a range of velocities from 450 to 575 fps, a range of angle of yaw from 0 to $\pm 45°$, and immersion from 2.5 to 5 inches.

From authors' summary by George Rudinger, USA

3688. Pankhurst, R. C., and Pearcey, H. H., Camber derivatives and two-dimensional tunnel interference at maximum lift, *Aero. Res. Counc. Lond. curr. Pap.* 28, 5 pp., Apr. 1950.

Paper reviews briefly the induced flow theory as applied to an airfoil in a wind tunnel. Values of camber derivatives are discussed and inapplicability of usual theoretical values at high angles of attack is pointed out. Need for experimentally determined correction factors to theory in this range is discussed.

M. J. Thompson, USA

Thermodynamics

(See also Revs. 3618, 3676, 3710, 3712)

3689. Crépin, R., and Lévy, R., On the evolution of gas systems in chemical equilibrium (in French), *Mémor. Artill. fr.* 24, 3, 679–728, 1950.

By defining the entropy of components in terms of their enthalpy and the integral of vdp, the calculation of the thermodynamics of burning gases is simplified and allows a direct estimate of the temperature combustion. Gilbert W. King, USA

3690. Kestin, J., Influence of variable specific heats on the high-speed flow of air, *Aero. Res. Counc. Lond. curr. Pap.* 33, 15 pp., June 1950.

For gas flows with high Mach numbers and high stagnation temperatures (T_0), the assumption that the isentropic index γ or the specific heats are constant, leads to errors in computing the stream temperature (T). Author uses available thermodynamic data for air to prepare new tables and a chart for T as a function of M and T_0, taking into account the dependence of γ on T. He shows that error resulting from using constant $\gamma = 1.400$ reaches maximum of 370 F at $M = 2.2$ and $T_0 = 5000$ F abs.

H. G. Cohen, USA

3691. Dugger, G. L., Effect of initial mixture temperature on flame speed of methane-air, propane-air and ethylene-air mixtures, *Nat. adv. Comm. Aero. tech. Note* 2374, 30 pp., May 1951.

Flame speeds of methane-, propane-, and ethylene-air mixtures were determined as functions of mixture composition at initial mixture temperatures from 34 to 344 C by a Bunsen-burner method. The maximum (optimum mixture composition) flame speeds increase with temperature at increasing rates and are expressed by approximate equations of the type $U = KT^n$, where U is flame speed in cm/sec, T is initial mixture temperature in °K, and K and n are constants. For methane $K = 0.000780$ and $n = 1.89$; for propane $K = 0.00246$ and $n = 1.72$; and for ethylene $K = 0.00971$ and $n = 1.565$.

Both the thermal theory of Semenov [*NACA T.M.* 1026, 1942] and the diffusion theory of Tanford and Pease [*J. chem. Phys.* 15, 861–865, 1947] predict the observed effect of temperature on maximum flame speed within 20%. A linear relation exists between maximum flame speed and the calculated equilibrium radical concentration. J. Howard Childs, USA

3692. Mackenzie, K. J., and Boddy, J. H., Fuel testing in gas-turbines. Some effects of atmospheric humidity on the operation of combustion chambers used for fuel testing, *Aircr. Engng.* 23, 264, 40–42, Feb. 1951.

In an effort to determine the causes of inconsistency in performance data of single combustion-chamber gas-turbine units used for fuel testing, the effects of moisture content in inlet air on these data are discussed. Errors due to neglecting atmospheric humidity in the determination of the air-flow rate and of the combustion efficiency are calculated, and it is shown that a change in

specific humidity from 0.006 to 0.020 lb water per lb of dry air may involve a variation in the air-flow rate of an order of 3%, and the corresponding inaccuracy of the combustion efficiency may be as high as 4% of its calculated value. The first is due only to the change of air density, and the second to the change of enthalpy of water contained in humid air.

The influence of water vapor on the combustion process is also described. The most important effects are shown to be the decrease of combustion temperature and the increase of inlet-air velocity. The first of these causes a small decrease of air/fuel ratio corresponding to weak flame-extinction limit, and the second causes a comparatively larger increase of this limit, so that the net result is an increase of the limiting air/fuel ratio due to presence of water vapor in inlet air. Antoni K. Oppenheim, USA

3693. Stanitz, J. D., Analysis of the exhaust process in four-stroke reciprocating engines, *Trans. Amer. Soc. mech. Engrs.* **73, 3,** 319–326, Apr. 1951.

An analysis is made of the effect of engine design and operating variables on the effectiveness of the exhaust process of four stroke engines. A characteristic parameter is developed by means of which proper exhaust porting (area and timing) are to be determined. This "gas velocity parameter" has the form $\phi = V_D/2\ C_{max}A_e a_0$, where V_D is piston displacement volume; C_{max} flow coefficient at maximum valve lift; A_e nominal exhaust-valve area; a_0 speed of sound at valve opening.

A maximum value of $\phi = 0.325$ is recommended. Although discussers take exception to some of author's assumptions, fair agreement between author's conclusion and experiment is demonstrated. The parameter ϕ should prove useful to the designer in laying out exhaust valve areas, lifts, and opening angles.
Peter Kyropoulos, USA

3694. Merlin, M., Contribution to the study of prerefrigeration (in French), *Bull. Assn. tech. marit. aéro.* no. 49, 713–730, 1950.

The importance of precooling the hold for the maritime transport of fruit (bananas) is discussed. Graphs and equations are given for the distribution of temperatures as a function of time during precooling period and voyage. Lothar Meyer, USA

3695. Sears, F. W., An introduction to thermodynamics: The kinetic theory of gases and statistical mechanics, Cambridge, Mass., Addison-Wesley Press, Inc., 1950, x + 348 pp. $6.

The field of thermodynamics is not only difficult to define in view of many tacit associations, but any description is strongly affected by the viewpoint of the writer. There are found in the current literature three fairly sharply distinguishable patterns. These correspond to and aim to meet the needs of the physicist, chemist, and engineer. It is doubtful whether a unified treatment is possible or even desirable. Many tests have been written from the engineering viewpoint and a number from that of chemistry. However, treatments devoted specifically to thermodynamics by physicists have been few, though most of these have been of excellent caliber.

This tradition of quality and physical rigor in thermodynamics has been maintained admirably by the author in this latest edition. While ostensibly prepared as a basic course for electrical engineers, this book is suitable for the broadest range of application. It will be found useful not only to those whose interest is exclusively in physics but also to any who believe that the thermodynamic training of engineers should be firmly grounded in physics.

The first nine chapters treat the fundamentals of thermodynamics as contained in the equation of state, the first and second law, and associated energy functions. Equations of state of any variety are presented only briefly; however, the thermodynamics of phase relations is developed in extensive detail. In this respect, emphasis is on fundamental thermodynamic analysis rather than on application to specific substances or processes. Thus, the Maxwell equations and associated partial differential relations are developed and applied to systems involving not only mechanical but also electrical and magnetic work. Entropy is introduced conventionally by references to the Carnot cycle and the Clausius inequality.

Chapter 10, referring to engineering applications of thermodynamics, is devoted almost exclusively to vapor cycles. In no place in the text are the Otto, Diesel, or Brayton cycles mentioned.

The last eight chapters are concerned specifically with the microscopic thermodynamic aspect of substances. These involve primarily the essentials of kinetic theory and statistical mechanics as applied to thermodynamic systems. The treatment would be advanced for the average engineering undergraduate, but represents a compact and coherent introduction for the physicist or graduate engineer.

For the theoretical physicist, this material is on an elementary level; for the engineer, it presents clearly the basic tools required for a physical interpretation of thermodynamic phenomena. In general, it represents a major addition to the literature in the field.
Courtesy of Journal of Applied Mechanics N. A. Hall, USA

3696. Newkirk, J. B., Smoluchowski, R., Geisler, A. H., and Martin, D. L., Phase equilibria in an ordering alloy system, *J. appl. Phys.* **22, 3,** 290–298, Mar. 1951.

The equilibria between phases in platinum alloys containing 42, 48, and 54 atomic per cent cobalt were determined below 1000 C. These systems show an order-disorder transition, the disordered phase being face-centered cubic, the ordered one tetragonal in analogy to the system Cu-Au. However, there exists a two-phase region where the ordered and disordered phase are coexisting. The system exhibits, therefore, a true phase transformation in an ordering alloy. A revised phase diagram is given, based on x-ray diffraction, electric conductivity, and microstructure analysis. Lothar Meyer, USA

3697. Guggenheim, E. A., and McGlashan, M. L., Corresponding states in mixtures of slightly imperfect gases, *Proc. roy. Soc. Lond. Ser. A,* **206,** 1087, 448–463, May 1951.

Although the principle that gases show corresponding states was developed in connection with the Van der Waals equation of state, it shows no reference to any particular equation of state when correctly formulated. Use of the principle as an independent criterion leads to a generally applicable method of calculating second virial coefficients of mixtures from the coefficients of components. Coefficients for several mixtures, as calculated by the authors, agree well with published experimental values, usually within less than 1 cm^3/mol.

The second virial coefficient B is defined as the limiting value of the expression $[V_m - RT/P]$ as the pressure approaches zero, where V_m is molar volume, R is the universal gas constant, T is absolute temperature, and P is absolute pressure. Then, from the principle of corresponding states, $B/V^* = \phi(T/T^*)$, where ϕ is a function applicable to all gases in corresponding states, while V^* and T^* are characteristic constants with different values for different gases. For most substances, V^* and T^* are the critical molar volume and critical temperature; but critical values for hydrogen and helium are corrected for quantum effects.

A table of characteristic constants and a plot of B/V^* against T/T^* are presented. In the latter, the relationship for all gases

can be approximated by a single smooth curve with an accuracy approaching but probably not equal to that of experimental measurements for individual gases.

The relationship between B and T is similar in form to that between interaction energy of and distance between individual molecules, namely, $\epsilon/\epsilon^* = \mu(r/r^*)$, where ϵ is interaction energy, r is distance between molecules, ϵ^* and r^* are constants characteristic of all molecules of a single kind, and μ is a function applicable to all kinds of molecules present. Analogies permit the deduction that the characteristic values of mixtures are determined by the equations

$$(V^*_{mix})^{1/3} = {}^1/_2(V_a{}^*)^{1/3} + {}^1/_2(V^*_b)^{1/3} \text{ and } T^*_{mix} = (T_a{}^* \cdot T^*_b)^{1/2}$$

where subscripts "mix," a and b refer to values for mixture, first component, and second component, respectively. The relationships as such are not new, but the basis therefore has been changed from postulated and possibly inaccurate equations of state to a principle applicable to gases in general. Hence, they acquire a theoretical validity much greater than that of other relationships derived or postulated on other bases. This theoretical validity is borne out by comparison of calculated values with experimental values for a number of binary mixtures of various components.

The dependence of the above relationships on the principle of corresponding states has further significance. The Lennard-Jones formula for calculation of B conforms to this principle and provides satisfactory values. Simpler approximations from two-parameter reduced equations of state, such as those of van der Waals and Berthelot, are seriously in error. Three-parameter equations can give good approximations, particularly if the parameters are properly chosen. As illustration, it is pointed out that B-T data for methane and n-butane are correlated better by parameters based on the principle of corresponding states than by the parameters the original authors chose [Beattie and Stockmayer, *J. chem. Phys.* **10**, p. 473]. Further, thep arameters based on this principle are shown to be considerably better for use both in extrapolation of measurements on these gases and for calculation of the properties of their mixtures. Jack D. Bush, USA

3698. Cox, R. T., The statistical method of Gibbs in irreversible change, *Rev. mod. Phys.* **22**, 238–248, 1950.

Author obtains a number of properties of systems not in equilibrium by using the methods of statistical mechanics. He considers two "loosely coupled" systems in various states, the second being regarded as the environment of the first. He discusses approach to equilibrium, viscous forces, recovery from a sudden disturbance of equilibrium, changes in an isolated system, minimal principles in isolated systems. Although there are many equations in the paper, the methods of obtaining them and of passing from one to another are of the nonmathematical, heuristic type usual in physical works on statistical mechanics.

C. Truesdell, USA

3699. Münster, A., Statistical mechanics of phase changes. I (in German), *Z. Naturforsch.* **6a**, 3, 139–151, Mar. 1951.

In order to eliminate uncertainties still obscuring the statistical theory of first-order phase transitions, the author, following Gibbs, attempts to derive statistical analogs to all the necessary thermodynamical quantities. He demonstrates the compatibility of the conventional thermodynamical treatment with the principles of statistical mechanics. He concludes that some details in Mayer's theory of condensation are wrong and that melting-point pressure curves must—similarly to boiling-point pressure curves—end in a critical point. These conclusions depend on the extension to infinity of relations applying originally to assemblies of a finite number of molecules. In reviewer's opinion, the argument connected with that extension involves a fallacy so that there is not sufficient foundation for the conclusions. Recent investigation [Holland, Huggill, Jones, and Simon, *Nature* **165**, p. 147, 1950] of the melting curve of helium does not confirm the author's assertion. R. Eisenschitz, England

Heat and Mass Transfer

(See also Revs. 3455, 3456, 3578, 3602, 3625, 3686)

3700. Podolsky, B., A problem in heat conduction, *J. appl. Phys.* **22**, 5, 581–585, May 1951.

Paper deals with problem of determining steady-state distribution of temperature in a homogeneous, isotropic, infinitely extended fluid moving with constant velocity, when heated by a periodically varying point source of heat. Solution is constructed from corresponding solution of problem in a stationary fluid. Much ingenuity is exhibited in obtaining solution, but, in general, the mathematics is straightforward.

Author concludes that the phase velocity may be assumed equal to the fluid velocity, with an error of less than 0.1%, even when the phase velocity in a stationary fluid is as large as 20% of the fluid velocity. William A. Wolfe, Canada

3701. Langstroth, G. O., Diehl, C. H. H., and Winhold, E. J., The evaporation of droplets in still air, *Canad. J. Res. Sec. A.* **28**, 6, 580–595, Nov. 1950.

The evaporation of 1- to 2-mm diam droplets into still air was studied for nine liquids including toluene, *n*-propyl alcohol, glacial acetic acid, ethylbenzene, *o*-chlorotoluene, *n*-amylalcohol, *n*-butylalcohol, and water. Experimental results agreed with theoretical calculations based on assumptions of applicability of method of quasi-stationary states and mass transfer by diffusion without convection. Convection currents were minimized by suspending droplets inside small, spherical evaporation vessels suspended in isothermal baths maintained at approximately 20 C for most of work. Droplet-surface temperature was found to be lower than its bulk temperature. Robert Mindak, USA

3702. Gol'denberg, S. A., Turbulent transfer in processes of heat exchange, diffusion and chemical processes (in Russian), *Izv. Akad. Nauk SSSR Otd. tekh. Nauk*, no. 5, 689–694, May 1950.

In a previous work [title source, no. 4, 1950] author defined two nondimensional coefficients $\varphi = \bar{\epsilon}/(\tau_0/\rho)^{1/2}(d/2)$, and $\psi = \epsilon/Ud$ (d diameter of tube, U mean velocity, τ_0 shearing stress at the wall, ρ density, and $\bar{\epsilon}$ coefficient of turbulent or apparent "viscosity" averaged over the tube cross section). From measured distributions of ϵ over the cross section (work by Nikuradse) he obtained relations between φ, ψ, and Reynolds number. He now proceeds to demonstrate the usefulness of his coefficients by relating them to the Nusselt number of turbulent heat transfer and the corresponding number for mass diffusion. However, since $Nu/Pé$ can be expressed as a function of the ratio φ/ψ, the averaged coefficient $\bar{\epsilon}$ thus being cancelled, reviewer is at a loss to see the conclusiveness of this demonstration. The well-known result is indeed quite independent of the averaging process.

A. von Baranoff, France

3703. Surinov, Yu. A., Analysis of some basic notions and problems of the theory of heat radiation (in Russian), *Izv. Akad. Nauk SSSR Otd. tekh. Nauk*, no. 4, 543–567, Apr. 1950.

Lengthy discussion of different mathematical approaches to problem of heat radiation: Through a rigorous integral equation,

through a simplified form of the integral equation, and by means of a system of algebraic equations, according to whether the physical properties of the radiating bodies are supposed variable from point to point or constant in certain regions.

A. von Baranoff, France

3704. Surinov, Yu. A., Concerning some fundamental equations of the theory of heat radiation (in Russian), *Dokladi Akad. Nauk SSSR (N.S.)* **72**, 469–472, 1950.

Let $B(M, s, \tau)$ be the intensity of radiation at point M, for time τ, and in the direction s. Author defines the vector spherical radiation $\mathbf{E}_{4\pi}(M, \tau) = \int_{(4\pi)} B d\omega$, where $d\omega = \mathbf{r}_1 d\omega(M, s)$ is a vector element of the solid angle ω, and $\mathbf{r}_1$ is a unit vector in the direction s. In the stationary case, which author considers almost exclusively, he shows that div $\mathbf{E}_{4\pi}(M) = 0$, and continues with discussions of the hemispherical vector radiation, $\mathbf{E}_{2\pi}(M, n) = \int_{(2\pi)} B d\omega$, and the projections, $(\mathbf{E}_{4\pi}, \mathbf{n}_1)$ and $(\mathbf{E}_{2\pi}, \mathbf{n}_1)$, on lines from M normal to a boundary surface F. He arrives at an integral equation for the radiation at F, given the temperature field, the configuration of the system, and the optical constants.

R. E. Gaskell, USA

3705. Alad'ev, I. T., Mikheev, M. A., and Fedínskiĭ, O. S., Dependence of heat exchange in pipes on the direction of heat flow and free convection (in Russian), *Izv. Akad. Nauk SSSR Otd. tekh. Nauk*, no. 1, 53–67, Jan. 1951.

Introducing the dimensionless Nusselt, Reynolds, Prandtl, and Grashof numbers, authors express Nu in terms of Re, Pr, and Gr. For $Re_l = 10{,}000$, the relation is $Nu_l = 0.028\ Re_l^{0.8}\ Pr_l^{0.4}\ (Pr_l/Pr_w)^{1/4}$ where the subscripts l and w refer, respectively, to the liquid in the tube and the wall. For smaller values of Re_l the relation is more complicated and Nu depends also on Gr. The tube was 60 cm long and 10.2 mm in diam. The velocity of the liquid ranged from 1.6 to 400 cm/sec and the temperatures ranged from 6 to 100 C.

D. Ter Haar, Scotland

3706. Kanzaki, H., On the recovery process of cold-worked metals, *J. phys. Soc. Japan* **6**, 2, 90–94, Mar.-Apr. 1951.

Results are given of specific heat measurements of cold compressed polycrystalline copper (99.8% purity), silver (99.9%), and aluminum (99.9%) during heating at 1 C per minute. Curves of specific heat vs. temperature exhibit two "valleys" which author associates with two processes having different activation energies; the first, recovery from lattice distortion, the second, recovery from grain fragmentation. Discussion is in terms of reaction rate theory.

George V. Smith, USA

3707. Datzeff, A., On heat propagation in a multi-layer medium (in French), *Godishnik, Sofia* **45**, 1,63–91, 1949.

Consider a body composed of several homogeneous layers with the surfaces separating the layers being parallel planes. The problem of heat propagation in this body is equivalent to the corresponding problem in a thin bar A which is composed of n homogeneous parts $A_i (i = 1, 2, \ldots, n)$ of length l_i, the lateral surface of the bar being impermeable to heat. If the bar is thought of as extending along the x-axis such that the section A_i lies in the interval (x_{i-1}, x_i), the temperature $u_i(x, t)$ of any such section satisfies the equation [1] $a_i^2 \partial^2 u_i/\partial x^2 = \partial u_i/\partial t$, where $a_i^2 = k_i/(\rho_i \sigma_i)$ with ρ_i, σ_i, and k_i being the density, specific heat, and thermal conductivity, respectively. The initial temperature is given as a function $\Phi_i(x)$ such that [2] $u_i(x, 0) = \Phi_i(x)$, $x_{i-1} < x < x_i$. The temperatures at the extremities O_0, O_n of A are given by the functions $\phi_0(t)$ and $\phi_n(t)$: [3] $u_1(x_0, t) = \phi_0(t)$, $u_n(x_n, t) = \phi_n(t)$, $t > 0$. At the points O_i common to two adjacent segments, the following conditions hold: [4] $u_i = u_{i+1}$, [4′] $k_i \partial u_i/\partial x = k_{i+1} \partial u_{i+1}/\partial x$, $x = x_i$, $t > 0$.

The problem is to find a set of functions $u_i(x, t)$ $(i = 1, \ldots, n)$ satisfying equations [1], initial conditions [2], and conditions [4] and [4′] at the points $O_1, O_2, \ldots, O_{n-1}$ with u_i and u_n satisfying conditions [3]. Let $\phi_i(t)$ indicate the temperature at the point O_i and consider the functions ϕ_i as known; the temperature $u_i(x, t)$ with initial value $\Phi_i(x)$ has the values $\phi_{i-1}(t)$ and $\phi_i(t)$ at the extremities O_{i-1}. Then u_i may be written [5] $u_i(x, t) = V_i(x, t) + W_i(x, t)$, where [6] $V_i(x, t) = \int_{x_{i-1}}^{x_i} \Gamma_i(x, \xi, t) \Phi_i(\xi) d\xi$,

$$\Gamma_i(x, \xi, t) = \frac{1}{2l_i}\left[v_{3i}\left(\frac{x-\xi}{2l_i}, \frac{a_i^2 t}{l_i^2}\right) - v_{3i}\left(\frac{x-2x_{i-1}+\xi}{2l_i}, \frac{a_i^2 t}{l_i^2}\right)\right] \quad [7]$$

and

$$v_3(x, t) = (\pi t)^{-1/2} \sum_{k=-\infty}^{\infty} \exp\left[-(x+\nu)^2/t\right] = 1 + 2\sum_{\nu=1}^{\infty} \cos 2\pi\nu x \cdot \exp(-\pi^2\nu^2 t) \quad [8]$$

where v_{3i} represents the function v_3 on the interval (x_{i-1}, x_i). The function $W_i(x, t)$ is given by

$$W_i(x, t) = -\frac{a_i^2}{l_i}\phi_{i-1}(t)^* \frac{\partial v_{3i}}{\partial x}\left(\frac{x-x_{i-1}}{2l_i}, \frac{a_i^2 t}{l_i^2}\right) + \frac{a_i^2}{l_i}\phi_i(t)^* \frac{\partial v_{3i}}{\partial x}\left(\frac{x_i - x}{2l_i}, \frac{a_i^2 t}{l_i^2}\right) \quad [9]$$

where the asterisk indicates the following convolution: [10] $\phi(t)^*(t) = \int_0^t \phi(\tau)\psi(t-\tau)d\tau$. The function [5] satisfies conditions [2] and the conditions [11] $u_i(x_{i-1}, t) = \phi_{i-1}(t)$, $u_i(x_i, t) = \phi_i(t)$. The functions u_i satisfy conditions [4] by the way in which they were constructed since both u_i and u_{i+1} become $\phi_i(t)$ for $x = x_i$. In order to show that [4′] is satisfied it is necessary to differentiate [5]. This gives an expression which is indeterminate at $x = x_i$, so in order to avoid this difficulty [4′] is replaced by the integral condition: (4″) $k_i \int_0^t (\partial u_i/\partial x)d\tau = k_{i+1} \int_0^t (\partial u_{i+1}/\partial x)d\tau$, which expresses the equality of the quantities of heat which pass through the point O_i from right to left during the time t. This leads to a system of $n-1$ Volterra integral equations of the first kind for the unknowns $\phi_i (i = 1, 2, \ldots, n-1)$. Having solved this system of equations, the results may then be substituted into the appropriate equations to give the set of functions $u_i(x, t)$ which constitute a solution of the problem. It is shown that this solution is unique. The stationary state problem is treated as a special case in which the $n-1$ integral equations of the preceding reduce to $n-1$ linear equations. If the end section A_n is allowed to become infinite in length while the other sections remain finite in length, the above system of integral equations become a system of Volterra integral equations of the second kind.

Courtesy of Mathematical Reviews. C. G. Maple, USA

3708. Kazakova, E. A., On the maximum heat flow when water boils under high and super high pressure (in Russian), *Izv. Akad. Nauk. SSSR Otd. tekh. Nauk*, no. 9, 1377–1387, Sept. 1950.

In the discussion following the paper of Farber and Scorah on "Heat transfer to water boiling under pressure" [AMR **1**, Rev. 927] several questions were raised regarding the variation of maximum heat flux and film coefficient. By reporting independent and carefully conducted tests, this report extends and clarifies these results. Tests were carried out up to 200 atm with film temperature differences in some cases up to 250 C. Smooth variations in maximum heat flux are obtained, showing a maxi-

mum equal to four times the maximum at 1 atm at about 75 atm. Results are shown graphically with no test data reported. The experimental procedure is described in detail. Reference is made to related work of G. H. Kruzhilin [AMR 1, Rev. 749] and E. A. Kazakova [*Dokladi Akad. Nauk SSSR* 61, no. 1, 1950].

Newman A. Hall, USA

3709. Hull, R. A., Wilkinson, K. R., and Wilks, J., The specific heat of liquid helium at temperatures between 0.6° and 1.6° K, *Proc. phys. Soc. Lond. Sec. A*, 64, part 4, 376 A, 379–388, Apr. 1951.

A detailed account is given of experiments made to measure the specific heat of liquid helium between 0.6 and 1.6 K. Below 1.4° the specific heat may be expressed as $C = 0.024\ T^{6.2}$ cal/gm/deg. Some implications of this result are discussed briefly.

From authors' summary

3710. Ellion, M. E., New technique for obtaining heat-transfer parameters of the wall and combustion gas in a rocket motor, *Trans. Amer. Soc. mech. Engrs.* **73,** 2, 109–114, Feb. 1951.

Gas side-wall temperatures are considered in the evaluation of various alloys for rocket-motor construction. Cylindrical specimens were inserted into the throat of a thick-walled nozzle. Heat flows and wall temperatures were determined by employing the nozzle as a heat meter with the temperature variation of the thermal properties of the material as the basis for calculations. A heat-flow meter for measuring the combustion-gas film coefficient is suggested. This meter uses the variation in thermal conductivity of two different alloys as the measuring scheme.

Byron E. Short, USA

3711. Werner, A. C., Two calorimeters for industrial measurements, *Rev. sci. Instrum.* **22,** 4, 245–257, Apr. 1951.

Two calorimeters for the determination of the heat capacity of solid materials and heats of fusion are described. Typical data for potassium nitrate and for a solution of nicotinic acid in sulfuric acid are presented. From author's summary

3712. London, A. L., and Kays, W. M., The gas-turbine regenerator—the use of compact heat-transfer surfaces, *Trans. Amer. Soc. mech. Engrs.* **72,** 5, 611–621, July 1950.

Paper illustrates the application of high-rating compact surfaces of the plate-fin-type to the regenerator-design problem for typical gas-turbine cycles. Comparisons are made with designs employing circular tube surfaces one inch and one-quarter inch in diameter. From authors' summary

Acoustics

3713. Ernsthausen, W., The rotating airfoil as a radiation problem, *Z. angew. Math. Mech.* **31,** 1/2, 20–35, Jan./Feb. 1951.

The sound field of a rotating propeller is equivalent to a periodically varying field of sources and doublets on the circular disk swept by the blades. Frequency corresponds to number of blades and angular velocity, phase to location of blades. For a standing propeller, velocity potential satisfies the wave equation. Thickness distribution of blades determines normal velocity and, hence, generates a field of sources (zero-order radiator); circulation around blades (determined by angle of incidence) is considered as imposing a force on surrounding gas and corresponds to a doublet distribution, which, in incompressible flow, yields thrust of propeller. Considering losses by sound radiation (calculated by author and others in prewar papers), total force on the blades is found from the local pressure distribution yielded by the source and doublet distribution.

The "mass" resistance and "radiation" resistance (imaginary part of force) can be expressed by dimensionless functions of the variable $kR = \omega R/c$ (ω frequency, R radius of the disk, and c sound velocity). These functions are calculated for the source distribution and compared with the frequency characteristic of the impedance of a simple mechanical substitution scheme, consisting of a mass with a friction and spring. By a suitable choice of mass, spring, and friction constants the characteristic of the scheme can very well be adapted to the calculated functions. For the doublet distribution, the radiation losses are calculated by integration over a large sphere. A mechanical impedance again can be adapted to the calculated resistance characteristic; here, however, the friction coefficient is taken as dependent on frequency. For this scheme the mass force is calculated, which appreciably shortens the work involved. Finally, combining these results, thrust coefficient can be found as function of kR. In compressible range (kR small) thrust increases proportionally to square of velocity, its downfall after a certain maximum value resembles resonance effects, and with still increasing velocity all energy is radiated as sound energy. It should be remarked that the theory is based on the Kirchhoff approximation and does not take account of secondary effects. For exact theory in case of standing circular disk, see Meixner, J., and Fritze, U., "Das Schallfeld in der Nähe der freischwingenden Kolbenmembran," *Z. angew. Physik.* I, 535–542, 1949. R. Timman, Holland

3714. Srivastava, A. M., Sound propagation in gelatinous substances (in German), *Kolloid Z.* **119,** 2, 73–85, Nov. 1950.

The velocity of sound in gelatinous substances is deduced. It is assumed that the solid particles in the fluid medium are solids of revolution about the $\theta = 0°$ axis, of the form given by $r = c \sin p\theta$. An expression for the velocity potential is deduced which is a more general version of the Rayleigh scattering law, to which it will reduce when it is assumed that the particles are spherical. One of the conclusions is that the scattered wave also depends on the form of the particles. Hitherto, other authors have assumed that the form of the particles was immaterial. The mathematical formalism involves solving the equations for the motion inside and outside the solid particles using a series of zonal harmonic functions. Albert London, USA

3715. Kronig, R., and Thellung, A., On the theory of the propagation of sound in He II, *Physica* **16,** 9, 678–690, Sept. 1950.

An extended calculation has been made of the types of waves that can exist in helium II. In addition to the first and second sound, authors find that two other types of waves can be transmitted. One of these is the ordinary viscosity wave present for liquids, which is damped out in a distance of the order of the wave length; the other, a wave aperiodic in space with exponential damping, which reduces it to small proportions in a distance of 10^{-7} m or less. All four waves are derivable from velocity potentials and are necessary to satisfy boundary conditions. The behavior of these waves is discussed in the region between the λ point and absolute zero. The theoretical calculations are compared with the experimental results of Pellam and of Maurer and Herlin. Warren P. Mason, USA

3716. Beatty, R. E., Jr., Boundary layer attenuation of higher order modes in rectangular and circular tubes, *J. acoust. Soc. Amer.* **22,** 6, 850–854, Nov. 1950.

Sound attenuation in ducts has interest because of noise problems in ventilating systems, etc. Author gives theoretical results for attenuation in rigid-walled tubes due to viscous and thermal losses in the "acoustic boundary layer" at the walls. Calculations are based on Cremer's boundary-layer admittance

concept, and, in the circular case, by an extension of this based on an electromagnetic wave-guide method.

Robert W. Morse, USA

3717. **Mikhaĭlov, I. G., and Gurevich, S. B., Absorption and velocity of ultrasonic waves in some very viscous liquids and amorphous rigid bodies** (in Russian), *Zh. eksp. teor. Fiz.* **19**, 3, 193–201, 1949.

Results are given of measurements on the absorption of ultrasonic waves in fused and solid rosin in the temperature range 40 to 145 C, and also on the absorption and velocity in polymethylmethacrylate. It has been established for the range of frequencies investigated and for high values of viscosity that, in agreement with relaxation theory, the absorption coefficient in rosin decreases with increase in viscosity. Investigations on the dependence of the coefficient of absorption on the frequency for rosin of high viscosity and for polymethylmethacrylate showed, however, that the coefficient of absorption decreases approximately in proportion to the square root of the frequency, which is not in agreement with relaxation theory in its simplest form. The frequency dependence of the coefficient of absorption found for polymethylmethacrylate is not accompanied by dispersion. It is suggested that the experimental results that have been obtained can be explained by the latest developments of the relaxation theory.

From authors' summary by Marie Goyer, England

3718. **Keck, W., Heller, G. S., and Williams, A. O., Jr., Measurements of the underwater sound field generated by quartz transducers,** *J. acoust. Soc. Amer.* **23**, 2, 168–172, Mar. 1951.

Narrow acoustic beams, at about 1 mc, are swept across a fixed microphone by rotation of the emitting crystal. Curve of angular variation appears directly on an oscilloscope; source-receiver distance is varied, to about 3 ft maximum. Beam patterns are rapidly explored. Axial distribution from X-cut crystals agrees with theory of simple piston of suitably chosen area in stationary baffle.

Philip Rudnick, USA

3719. **Matta, K., and Richardson, E. G., A hot-wire ultrasonic interferometer and its application to vapors,** *J. acoust. Soc. Amer.* **23**, 1, 58–61, Jan. 1951.

An ultrasonic interferometer with a fixed path is described. The amplitude for the sound waves is measured by a hot-wave detector with a 0.001-in. Pt-wire, calibrated by a suitable method. In use, the interferometer has such a position that the wire is vertical, avoiding convections by the hot wire. About 20 nodes between source and reflector are measured by the moved wire with a constant distance between source and reflector. For diffraction of the beam, a correction is used. Absorption coefficients and sound velocities are measured for vapors of ethyl-ether, ethylene, benzene, carbon disulphide, and acetaldehyde at a temperature slightly higher than the boiling point of the liquids. The pressure was slightly above atmospheric. The frequencies were 59.5, 87.6, 199.6, 400.7, and 688.6 kc/sec. The latter three of the substances showed some dispersion with frequency, while the sound velocity of the other vapors was constant within the frequency range. All vapors show an absorption considerably greater than the absorption calculated by the Stokes-Kirchhoff equations.

O. Ruediger, Germany

3720. **Westervelt, P. J., Acoustical impedance in terms of energy functions,** *J. acoust. Soc. Amer.* **23**, 3, 347–348, May 1951.

Author develops a mathematical procedure for deriving expressions for acoustic impedances from energy considerations and applies it to a spherically symmetric source, a small piston in an infinite baffle, and to the radiation mass of an orifice in a circular tube.

C. W. Kosten, Holland

Ballistics, Detonics (Explosions)

(See also Rev. 3689)

3721. **Palechek, E. M., On the question of calculation of trajectories of the centers of gravity of artillery shells** (in Russian), *Prikl. Mat. Mekh.* **15**, 1, 83–96, Jan.-Feb. 1951.

The essence of Kazakov's method [*Prikl. Mat. Mekh.* **9**, 129–138, 1945] for numerical integration of the system of differential equations of the particle problem in exterior ballistics lies in the fact that the extrapolations are not directly performed upon the one or the other variables, but rather upon certain auxiliary quantities which have little influence on the results of extrapolation. Kazakov himself used as the independent variables the abscissa of the center of gravity and the time of flight of the particle.

Author extends Kazakov's method to the case where either the tangent of the angle of inclination of the tangent to the trajectory or the horizontal component of the velocity is taken as the independent variable. Estimates of the relative error of the results for each choice of the independent variable are given, and the appropriateness of the choice to the purpose is discussed.

E. Leimanis, Canada

3722. **Johansson, C. H., and Langefors, U., Conditions for precision measurements of the speed of detonation by the Dautriche method,** *IngenVetenskAkad. Tidsk. Tekn. Forsk.* **22**, 1, 1–9, 1951.

It is shown how to achieve a simpler experimental procedure and a greater accuracy than before by using the Dautriche method for determining the speed of detonation in explosives forming cylindrical bars.

F. Schultz-Grunow, Germany

Soil Mechanics, Seepage

3723. **Piskunov, N. S., Solution of a boundary problem for a nonlinear parabolic equation of the motion of liquids and gases in a porous medium** (in Russian), *Doklady Akad. Nauk SSSR (N.S.)* **76**, 4, 505–508, Feb. 1951.

Paper considers an equation of nonsteady flow of fluids in a porous medium

$$\partial^2 p^2/\partial x^2 + \partial^2 p^2/\partial y^2 = a^2 \partial p/\partial t$$

and it is shown that when p is a function of r and t only, and some additional conditions are present, a solution of this equation can be found by reducing it to an integral equation, which can be solved approximately.

A. Hrennikoff, Canada

3724. **Christensen, C. B., Geostatic investigations,** *Ingen-Vidensk. Skr.* no. 3, 128 pp., 1950.

Paper deals with stress conditions within embankments and their subsoil caused by the weight of fill and any external loading, oblique or vertical. Author develops fundamental formulas for vertical and horizontal stresses produced by a line load applied to embankment section. Assuming the law of superposition, he integrates these fundamental expressions to obtain stress distribution diagrams for a variety of loading. This development is mathematical but author has attempted to correlate it with experimental results obtained by earlier investigators.

Illustration is made of the use of stress diagrams to investigate the stability of embankments, stability of subsoil beneath em-

bankments, and to compute pressures on structures contained within the fill or retaining structures for the fill. Of particular interest and importance is the method used for stability computations which allows a determination of safety factor for any increment along the failure surface, thus indicating the possibility of a progressive failure.

Author recognizes the difficulties involved in computing pressures on retaining structures by this method because of the variety of possible yielding conditions.

Philip P. Brown, USA

3725. Kufarev, P. P., Solution of the problem of an oil-bearing contour for a layer with a chain of sinks (in Russian), *Dokladi Akad. Nauk SSSR (N.S.)*, **75**, 3, 353–355, Nov. 1950.

Author emphasizes use of elliptic functions to solve problem of the oil-bearing contour if oil initially fills the infinite strip $|Re\ zl < \omega_1{}^0/2$, containing an infinite chain of symmetrically placed sinks of equal strength. R. E. Gaskell, USA

3726. Polubarinova-Kochina, P. Ya., On the influence of the slope of the water bed and of infiltration on unsteady motion of ground water (in Russian), *Dokladi Akad. Nauk SSSR (N.S.)* **75**, 4, 511–514, Dec. 1950.

Problem of determining the height of the ground water is reduced to the solution of a linear partial differential equation with certain boundary conditions. Explicit solutions are given for various conditions which fall under the case mentioned in the title.

Courtesy of Mathematical Reviews H. P. Thielman, USA

3727. L. P. H., New equipment for soil compaction (in French), *Tech. mod. Constr.* **5**, 12, p. 372, Dec. 1950.

Two methods of soil compaction being used in Russia are described. The first method compacts by causing explosions to take place in a cylinder of a 500-kg or 100-kg compactor. The compactor rises 30 to 50 cms, then falls freely onto the soil, and the process repeats. It is guided by an operator who adjusts its position when it is in mid-air. The second method consists of a heavy slab which is lifted by a crane and dropped on the soil. Both methods seem suitable for compacting areas of limited width, such as railroad and highway embankments. The explosion method can compact the soil to a depth of 60 cms and the heavy slab method to a depth of 90 cms, compared with 25 cms by sheep's-foot roller. Eben Vey, USA

3728. Hoeppel, R. W., A continuous flow, high temperature viscosimeter for drilling muds, *Rev. sci. Instrum.* **22**, 4, 248–253, Apr. 1951.

A continuous-flow, tube viscosimeter is described which permits determination of viscosities and gel strengths of abrasive thixotropic fluids at temperatures up to 250 F. Provision is made for the introduction of treating chemicals into the closed system, while under pressure, without interrupting fluid flow. The instrument employs a special rubber-tube pump which maintains constant flow under various pressures and is unaffected by abrasive materials. Flow characteristics of the pump are discussed, and tube and Stormer viscosimeter data are compared.

From author's summary

3729. Slobod, R. L., Chambers, A., and Prehn, W. L., Jr., Use of centrifuge for determining connate water, residual oil, and capillary pressure curves of small core samples, *J. Petr. Technol. Trans.* **3** (192), 4, 127–134, Apr. 1951.

A rapid and accurate method is described which uses a standard centrifuge with a minimum of special equipment. By use of accelerations ranging up to 25,000 gravities, it is found possible to establish an accurate capillary pressure curve for most samples within a single day's work. Core holders are provided with pipettes that are read by means of a stroboscope. From 30 minutes to one hour is usually sufficient to establish equilibrium for any given rpm.

Determinations of connate water and residual oil check closely with results obtained by slower and more laborious methods. End butts placed in capillary contact with the core largely remove the zone of high water saturation from the specimen, giving greater accuracy of results. Harold H. Munger, USA

3730. Mandel, J., Essay on physical mechanics of pseudo-solids (in French), *Ann. Ponts Chauss.* **120**, 3, 245–312, May-June, 1950.

A systematic compilation of published and of original mathematical information on the movement of gas and/or liquid through beds of solid particles when either or both fluid phases are continuous, and on the mechanical effects of the liquid on the solid structure. Relations are shown to exist between the fraction of the void space which is filled with liquid and (1) the mean radius of curvature of the gas-liquid meniscuses, (2) the relative humidity in the gas, and (3) the capillary rise. It is shown that temperatures within a porous solid wet with water are not affected appreciably by vaporization and condensation within the pores.

Forces are transmitted partly by the solid-solid contacts, upon which all deformations of soils depend, and partly by the liquid surface-gas interface, upon which cohesion depends. The concept of contact constraint permits quantitative evaluation of the former, by a "contact tensor."

Illustrations are given of flow through an earth dam of triangular section, and of contraction of various shapes on drying. It is shown that the average deformation increases proportionally with the degree of drying. C. F. Bonilla, USA

3731. Tollenaar, D., and Blokhuis, G., The structure of paper, *Appl. sci. Res. Sec.* **A2**, 2, 125–141, 1950.

Data on striking-away, capillary rise, and weight of liquid absorbed by porous paper are inconsistent with an assumption of uniform pore radius. Assuming the number of pores of any given radius follows a normal Gaussian distribution, the standard deviation of the radius is introduced as an additional constant, and the above data can be explained. For rotary printing, paper with a flat distribution of pore sizes is preferable.

C. F. Bonilla, USA

Micromeritics

3732. Feifel, E., Particle shape and law of falling (in German), *Forsch. Geb. Ing.-Wes. Ausg. B.* **17**, 1, 21–27, 1951.

Aim of article is to focus attention on effect of appreciable divergences from spherical shape on behavior of small particles in cyclone-type separators and in settlers. Serious errors may arise from calculations based on assumed spherical shape and on particle-size distribution obtained by sieving. Author suggests measuring settling speeds of differently shaped particles at low Reynolds numbers (<1), since such data are of utmost interest.

[Reviewer's remarks: Obviously author is not acquainted with the work by Krumbein, McNown, and others on settling speeds of nonspherical particles in the viscous range (e.g., McNown, J. S., and Malaika, J., *Trans. Amer. geophys. Un.* **31**, 74–82, Feb. 1950). In view of the interest in data at low Reynolds numbers, the data given in table 1 on flow resistances of various bodies

without mention of corresponding Reynolds numbers are somewhat misleading, because these data apparently refer to high Reynolds numbers.] J. O. Hinze, Holland

Geophysics, Meteorology, Oceanography

(See also Revs. 3485, 3486, 3701)

3733. Scorer, R. S., On the stability of stably stratified shearing layers, *Quart. J. roy. meteor. Soc.* **77,** 331, 76–84, Jan. 1951.

The wave equation for two-dimensional shearing air currents is discussed with respect to the occurrence of stable and unstable wave forms. It is shown that stable waves can exist only if they travel at all levels with a speed different from that of the air. The formation of unstable waves may depend on the rate of shear and stability. A few hints are given on how one could proceed to determine the wave form in special cases. No detailed results are given. Horst Merbt, Sweden

3734. Miyazaki, M., A theory of the transversal velocity distribution in ocean currents, *Oceanogr. Mag. Centr. Meteor. Observ. Tokyo* **2,** 2, 53–57, June 1950.

3735. Verhoogen, J., The adiabatic gradient in the mantle, *Trans. Amer. geophys. Un.* **32,** 1, 41–43, Feb. 1951.

Author derives an expression by which the ratio of the thermal expansion coefficient to the constant-pressure specific heat can be computed from seismic data. Values of this ratio are tabulated for the case of olivine under normal conditions at depths between 200 and 2600 km. Temperatures at these depths are calculated in terms of the temperature at 200 km, using an expression involving the ratio and assuming reversible adiabatic change of pressure and hydrostatic equilibrium. The average adiabatic gradient between depths of 200 and 2600 km is found to be $1.7 \times 10^{-4} T_{200}$ deg/km. Donald M. Vestal, Jr., USA

3736. Craddock, J. M., Advective temperature change in the 1,000-700 mb and 700-500 mb layers, *Quart. J. roy. meteor. Soc.* 77, 331, 51–60, Jan. 1951.

It is shown, on the basis of about 700 measurements made at a particular station during six months, that the thermal change in the troposphere due to advection alone is largely offset by the other processes at work, so that the resultant change is a residual having a correlation of about 0.6 with the advective component.

The shear hodograph of an upper wind sounding is shown to afford little useful information on the resultant thickness tendency.

This study utilizes methods of analysis of upper air temperatures which have been used by researchers at New York University [Panofsky, H. A., *Bull. Amer. meteor. Soc.* **25,** p. 271, 1944; Fleagle, R. G., *J. Meteor.* **3,** 9–13, 1946; Fleagle, R. G., *Engng. Res. Div.*, New York University, Mar. 1946, (mimeographed); Miller, J. E., *Meteor. Pap.* **1,** 1, New York University 1948]. The close agreement of results in these independent studies is evidence of the reliability of the major conclusions. R. G. Fleagle, USA

3737. Charney, J. G., Fjörtoft, R., and von Neumann, J., Numerical integration of the barotropic vorticity equation, *Tellus* 2, 4, 237–254, Nov. 1950.

This is the third [for the second see AMR 4, Rev. 945] progress report by Charney and collaborators in a program whose ultimate objective is numerical weather forecasting by automatic calculating machine. The method is described fully for integrating the equation, using the ENIAC, in examples taken from actual weather situations. The resulting extrapolations, in time, of the situations were compared with the actual succeeding situations.

One can raise no enthusiasm for the results as forecasts, but they should not be judged as such. Considerable calculation time was required so that the forecast could only just keep pace with the weather, and the grid of points used was undesirably coarse; but these were limitations imposed by the size of the ENIAC and will not apply in future machines. Solution of the equation serves to advect the vertically averaged vorticity of the initial wind field, and expresses the results in terms of the height of an isobaric surface—the equivalent barotropic level. It is difficult to extract initial values from a real weather situation to give the vertical mean because the atmosphere is not barotropic. The authors describe the equation as one in a hierarchy of increasing complexity, there being no point in attempting the baroclinic case until it has been shown possible to solve the barotropic equations. It might have been more profitable, in that case, to work out the development of hypothetical initial wind fields and examine them for remarkable features rather than attempt to compare results with observations to which the equations did not properly apply. The detailed discussion of the actual cases is of little interest compared with the more general discussion that follows.

Regarding the problem of putting in smooth values from a weather chart when the grid is made finer (one aspect of the problem, discussed in the second paper, of eliminating noise), the authors' suggestion of fitting the data to polynomials or other mathematical functions of form chosen in advance seems fundamentally wrong because the computations do not require a functional form for the values, but only smoothness. This can be gained in other ways and should ultimately be done by the machine.

The problem of what boundary conditions to apply to the region cannot be solved by theory for such a complex equation; so the authors reasonably conclude that where air is leaving the region the tangential pressure gradient must be given, and where it is entering its vorticity must also be given. This lends point to reviewer's criticism of the second paper that disturbances propagated (like waves) through the atmosphere, as opposed to advected in it, are irrelevant, for the pressure gradient across the boundary is not required.

Baroclinic development can be assessed roughly from observations from the changes in area enclosed in isolines of absolute vorticity. These were, in the actual cases studied, considerable at times, and, since the equations are nonlinear, any development early in the forecast period will make the forecast progressively more in error as time proceeds. The authors consider the barotropic model valuable for the sometimes quite long intervals in which there is no baroclinic development. These intervals must be recognizable, however.

A simplified baroclinic model is discussed briefly, one assumption being that potential temperature is advected horizontally. This is found less in agreement with observations in the stratosphere than the assumption of isentropic flow. The analysis follows Sutcliffe's in many respects, and readers will look forward to publication of the results of the promised fuller application of numerical methods to it. R. S. Scorer, England

3738. Kanai, K., The effect of solid viscosity of surface layer on the earthquake movements, *Bull. Earthq. Res. Inst. Tokyo Univ.* **28,** parts 1–2, 31–35, Jan.–June 1950.

Paper investigates amplitudes of plane transverse seismic waves passing vertically upwards through a semi-infinite solid on which rests a finite superficial layer with plane horizontal boundaries.

Media are treated as imperfectly elastic of type in which rigidity parameter is replaced by the form $\mu + \mu'\partial/\partial t$, μ' being small. Amplitude-frequency curves of surface movement, as yielded by solution, are plotted for special value of μ' in superficial layer; authors claim good agreement with results yielded by accelerometers in Tokyo and Yokohama. K. E. Bullen, Australia

3739. Inoue, E., The application of turbulence theory to oceanography (in Japanese), *J. meteor. Soc. Japan* **28**, 420–424, Nov. 1950.

It is suggested from the modern theory of turbulence that the coefficient of turbulent diffusion K is proportional to $\epsilon^{1/3}L^{4/3}$, where ϵ is the mass rate of dissipation and L is the scale concerned [von Weizsäcker, C. F., AMR **3**, Rev. 1553]. The theoretical suggestion corresponds to the empirical relation found by Richardson for atmospheric turbulence, K (cm²/s) = 0.2 $L^{4/3}$, L in cm. Summarizing various existing observed results, author shows that an empirical formula K (cm²/s) = $0.01L^{4/3}$ holds for oceanic turbulence. He also explains the difference between the numerical constants of the atmospheric and oceanic turbulence by supposing that the oceanic turbulence is set up by the action of the shearing stress at the surface. Itiro Tani, Japan

3740. Nanniti, T., A lazy thermometer, *Oceanogr. Mag. Centr. Meteor. Observ. Tokyo* **2**, 2, 59–65, June 1950.

In order to measure the mean water temperature on a fishing ground, author devised a lazy thermometer which can be used attached to a trawl net. An ordinary stem thermometer is embedded in a glass bottle filled with sawdust, the bulb of the thermometer being situated at the center of the bottle.

The problem of heat conduction in this thermometer is treated theoretically for a cylinder of finite length, the thermal constants necessary for numerical computation being obtained by experiments. The lag of this thermometer can thus be computed by the theory. Author, after trying this instrument in the shallow water near the coasts and at the bottom of a fishing ground, concluded from comparison of the result with that computed that it is necessary, in order to get a reliable result, that this thermometer should be immersed at least 2 hours in the water and the reading should be taken as quickly as possible. If sufficient caution is taken, the maximum error in reading the temperature will not exceed 0.5 C, even when the difference of temperature between the water surface and the layer 100 m deep is as large as 15 C, and also when it takes 5 minutes to pull up the thermometer.

Koji Hidaka, Japan

Lubrication; Bearings; Wear

(See also Revs. 3505, 3590)

3741. Fishman, I. M., On the motion of a very viscous liquid between journal and bearing (in Russian), *Prikl. Mat. Mekh.* **14**, 6, 593–610, Nov.–Dec. 1950.

The hydrodynamical theory of the motion of a viscous liquid between journal and bearing results in a nonlinear equation, as is well known. In order to solve the problem, several simplifications are introduced, such as the assumptions of a constant viscosity, a very small velocity, and neglect of the inertia forces in the liquid. Author presents a mathematical analysis accounting for the inertia forces. He makes use of a method of iteration starting from the simplified solution (Sommerfeld resp. Joukowsky) without proving the convergency of the method. He explores the motions under a constant load, as well as under a periodically variable load having a constant direction.

W. Kochanowsky, Germany

3742. Clayton, D., An introduction to boundary and extreme pressure lubrication, *Brit. J. appl. Phys. Suppl.* no. 1, 25–32, Jan. 1951.

This comprehensive review on boundary and extreme pressure lubrication includes explanations of boundary friction, in particular, the adhesion theory. Dealing with the influence of the main parameters on friction values, velocity is considered first, together with explanation of friction-induced vibrations in terms of the main type of friction vs. velocity curve. A simple general statement on the effects of temperature and surface roughness on friction cannot yet be given. Brief references are made to the high temperatures reached at sliding contacts, and to electrical effects. In terms of these fundamental concepts, a sketch is given of the mechanism of running in, of wear (including wear-testing and anti wear additives), of the behavior of surfaces under severe conditions leading to scuffing (including the function of extreme pressure lubricants). Finally, the value of solid lubricants is mentioned, and also the benefits that can be obtained by chemical pretreatment of solid surfaces.

W. Kochanowsky, Germany

Marine Engineering Problems

3743. Telfer, E. V., Frictional resistance and ship resistance similarity, *Trans. Instn. nav. Archit. Lond.* **92**, 1, 1–15, Jan. 1950.

See AMR **3**, Rev. 2560.

3744. Kronberger, E., Further notes on stability, *Shipbuilder* **57**, 495, 104–105, Feb. 1950.

Addendum to earlier article [title source, **54**, p. 129, Mar. 1947] develops further relations connecting transverse moment of inertia of waterplane in inclined position (function of ship's angle of inclination) with moment of stability. Should be read in conjunction with original article. A. O. Williams, Jr., USA

3745. Boggis, A. G., and Corlett, E. C. B., An instrument for directly recording the metacentric height of a ship, *J. sci. Instrum.* **27**, 10, 278–279, Oct. 1950.

Instrument incorporates a pendulum for determining inclination of vessel; displacement of pendulum is measured by a lamp and photocell system. By suitable calibration, metacentric height is given directly in feet. Reviewer believes that an electrical system more reliable than a photocell unit could have been employed, and that claim of applicability of instrument while vessel is proceeding on voyage should be explained further.

Edward Wenk, USA